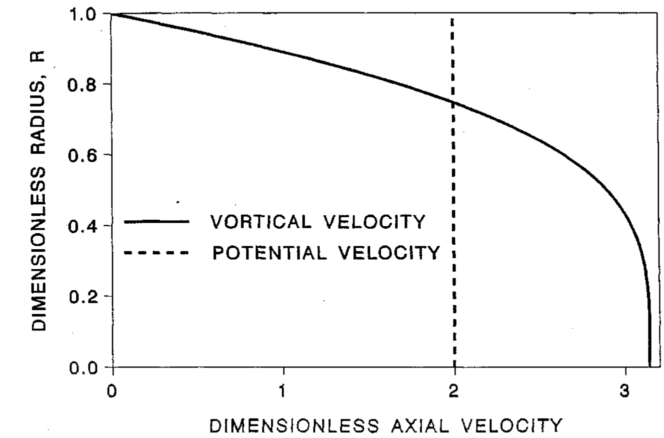

Fig. 1 Axial velocity profiles.

velocities are 36%[4] below the vortical predictions. Therefore, the vortical flow model is a superior flow model in both the laminar and turbulent regimes.

Culick's[4] paper gives both the rotational and irrotational solutions for the flow inside a cylindrical port motor with a constant-radius grain. Solid rocket motors that fly in the atmosphere have large length-to-diameter ratios (10–30 are typical values), and for purposes of illustration, the vortical and potential axial velocities are compared, herein, for a 10-to-1 geometry. As will be obvious, differences between the two solutions increase as the length-to-diameter ratio increases.

The dimensionless axial velocity profiles may be obtained easily[4] as

$$u/(V_bX) = 2 \tag{1}$$

for the potential flow, and

$$u/(V_bX) = \pi \cos(\pi R^2/2) \tag{2}$$

for the rotational flow. Where V_b is the flow velocity at the propellant burn surface, r_0 is the radius of the burn surface, and X and R are the dimensionless axial and radial coordinates normalized with r_0. The quantity X is measured from the motor head end. These velocity profiles are plotted in Fig. 1 and show significant differences both near the wall and on the axis. The largest discrepancy in the flow velocity occurs at $X = 20$, $R = 1$. At this point, the vortical solution gives a radial velocity of V_b and an axial velocity of 0; the corresponding values for the potential solution are V_b and $40V_b$. Thus, the potential flow has a velocity magnitude error of about 40 and a flow angle error of about 90 deg. Since the drag of large droplets varies as the velocity squared, the slag particle drag at that point is in error by a factor of about 1600 in magnitude and 90 deg in direction.

Various investigators[1–3,10] are using potential flow models to compute the gas flow in a two-step slag model. An additional problem with the potential flow approach is that users of these models usually assume some arbitrary vortex shape in the re-entrant region. It has been reported by Smith-Kent and Perkins[10] that this assumed shape has a large effect on the predicted slag accumulation. There is also evidence that, at least in some motors, this separation region is not nearly as large as is frequently assumed. Misterek et al.[11] computed the viscous and inviscid flow for the PAM motor and found the separated flow region to be so small that the viscous solution differed insignificantly from the inviscid solution. It is believed that this occurred in the motor considered in Ref. 11 because the flow in the re-entrant region is radially inward and therefore always experiences a favorable pressure gradient.

This Comment has shown that there is no theoretical basis for the potential flow approach. Comparisons with experimental data have shown good agreement with the analytical, vortical solution published by Culick.[4] Finally, some simple graphical and numerical comparisons of Culick's analytical, vortical, and potential solutions demonstrate that use of the latter can be expected to produce large errors in the slag particle trajectories and, hence, the predicted slag capture.

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Reply by the Authors to J. W. Murdock

D. Brent,* F. Fendell,† and P. Acosta‡
TRW Space & Electronics Group, Redondo Beach, California 90278

OUR objective in Refs. 1–3 is to estimate the slag accumulation (in the combustion chamber of a solid rocket motor with a metallized composite grain) during the burn, and especially to estimate the total slag accumulation at the end of the burn. The practical motivation is to suggest altered grain composition and/or initial grain configuration, to reduce the performance-degrading accumulation. We seek to replicate the flowfield in the motor during the burn only to the accuracy necessary for a practically useful estimation of the slag retention. Because we seek to avoid any nonessential

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*Program Manager, Center for Fluid Mechanics and Propulsion Technology. Member AIAA.

†Staff Scientist, Center for Fluid Mechanics and Propulsion Technology. Associate Fellow AIAA.

‡Member of the Technical Staff, Center for Fluid Mechanics and Propulsion Technology.

calculation, not an excuse to pursue computational fluid dynamics, we adopt what we believe to be the simplest reasonable flowfield model, consistent with estimating the trajectories of alumina particles of a wide range of sizes, virtually throughout the combustion chamber throughout the burn. Whether our potential-flow model is adequate for our objectives is decided only by comparing results of our analysis with observations. We offer evidence below that our analysis is fully adequate for our purposes.

As Murdock notes, we are aware of Culick's alternative flowfield representation,[4] which enforces a no-slip boundary condition at the burning grain for an inviscid fluid; we continue to regard this upgrade of the model as of low priority, relative to other, still-unexecuted upgrades that we discussed. Murdock cites a specific (exceptional) situation in which a locally large error might be incurred (more generally, the linear Stokes drag law adequately describes the interphase drag); however, he offers no evidence that adopting the alternative flowfield representation improves slag-retention estimates.

We digress to address several statements by Murdock, before concluding with a comparison of our model with data.

First, contrary to Murdock's characterization, we modeled the velocity field as a simple counterflow only near the head end of the motor, where such a model is suitable because the bore is locally a simple cylinder. However, every first- or second-stage solid rocket motor known to us has very significant slots cut into the aft end of the grain, to uniformize the efflux during the burn; much of the aft end may be grain-free during most of the burn.[5] Furthermore, we need to take account of the deeply submerged (i.e., deeply recessed) nozzle.[2,3] Thus, the cylindrical-bore-flowfield models adopted by Culick suffice uniformly only for very academic grain geometries, free of slots and a nozzle. While Culick's models may suffice for his objectives (estimating the damping of pressure oscillations), for our slag-retention-estimation objectives, we necessarily generalized, and chose to proceed as follows. We applied the continuity equation "in the large" (i.e., in integral form) in order to obtain the local axial velocity, then "in the small" (i.e., in differential form), in order to obtain the local radial velocity. This approach necessitated adoption of a radial profile for the axial velocity; we took the profile to be transversely uniform; the radial velocity then varies linearly with the radius. These approximate results were in excellent agreement with more meticulous, computational results obtained, relatively laboriously, by finite element solution (see below) of the same potential-flow formulation with the same Neumann-type boundary condition (i.e., with the velocity component perpendicular to the bounding solid surface assigned).

The outcome is that Culick's closed-form solutions, with or without tangential-velocity slip at the grain/gas interface, hold in practice only for the forward-end cylindrical bore (a region of limited concern, since, whether the flow has slip or not, forward-end-generated particles are not retained in the cavity[1-3,6,7]). Accordingly, Culick's solutions are inadequate for our purposes, except for our one preliminary, exploratory study,[1] which is dedicated entirely to the forward end. Parenthetically, we note that experimental studies[8,9] [if nitrogen, diffused through a porous matrix at roughly room conditions, simulates both the "injection" of fluid from a burning grain and also the equation of state (and hence the ratio of heat capacities) of that fluid in the combustion chamber] suggest that enforcing no slip at the boundary for an inviscid fluid yields a representation of the cylindrical-bore flow generally more accurate than the representation derived for potential flow. The distribution of vorticity across streamlines in an inviscid incompressible steady flow, whether two-dimensional or axisymmetric, can be assigned arbitrarily.[10-12] Since assigning the distributed vorticity to enforce the no-slip boundary condition for an inviscid fluid for a practically interesting chamber geometry is a nontrivial, unexecuted task, we cannot compare slag-accumulation results for slip and no-slip treatments.

Incidentally, the mass flux across any surface fixed in the coordinate system is necessarily continuous, whether or not a vortex sheet is coincident with that surface.[11,12]

We now turn to a comparison of results of our modeling with data from two very different solid-rocket motors, tested under very different conditions. The same quasisteady *potential-flow* formulation with Neumann-type boundary conditions yields the velocity field, for each of many times during the burn, and at each time the trajectories of spherical alumina particles, formed just off the grain boundary and transported by the irrotational flow, are computed with interphase drag, gravity, and flight acceleration (if appropriate). We determine the evolution of a locus that we call the "separatrix"[2,3] as follows. At each time, particles leaving the grain boundary downwind of the separatrix accumulate in the aft end of the combustion chamber as slag (these molten particles impact the motor case or backside of the deeply recessed nozzle, or become trapped in a recirculatory flow); particles leaving the grain boundary upwind of the separatrix enter the nozzle and exit the chamber.[2] From knowledge of the separatrix position in time, the slag accumulation in time follows straightforwardly. Standard assignments for the (pressure-dependent, log-normal, bimodal) particle-size distribution, the slag and gas densities, the gas viscosity, the chamber pressure, etc., are adopted, with the mass-mean-particle-size assignment to be discussed below. Meticulous solution for the *potential flow* is obtained by a finite element calculation, with irregularly shaped quadrilateral cells, and with a first-order Galerkin approximation, which is equivalent in accuracy to a second-order-central-difference scheme on a rectangular mesh. The finite element model has second-order accuracy on the interior physical space and on the boundaries. A set of piecewise interpolation functions spans a large network (1000–20,000) of four-cornered cells (side lengths and corner angles are free to be assigned), in the physical (and solution) space. A piecewise continuous isoparametric bilinear function is used to approximate the stream function throughout the solution space. The Galerkin-approximation-based integral equation is treated conveniently in a natural-coordinate space. A four-point Gauss-Legendre scheme is used to integrate the Galerkin integrals, and the resulting set of linearized equations is solved by locally accelerated successive over-relaxation.

Real-time radiometric video-movies of a 140-s-long static firing of the large-length-to-diameter Titan IV solid-rocket-motor unit were interpreted (by the Lawrence Livermore National Laboratory, issued May 3, 1993) to give the aft-end-slag-pool depth in time, and we averaged over the sloshing to infer the slag accumulation. If we increase the mass-mean particle size from the nominal value[13,14] of about 115–160

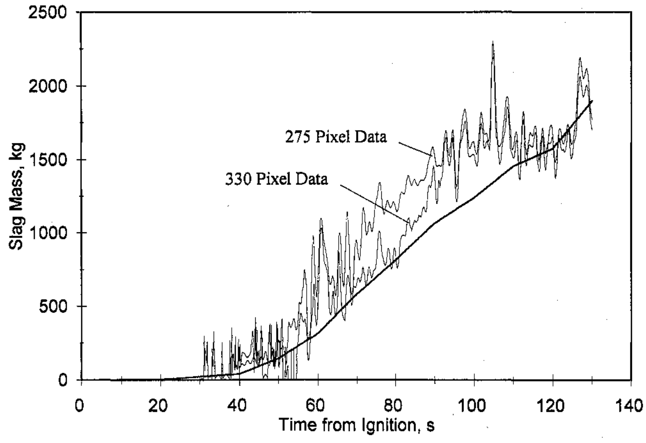

Fig. 1 In-chamber-accumulated-slag mass vs time from ignition: real-time X-ray (10-Mev-photon) data from a Titan IV QM2 static-test firing, and potential-flow predictions.

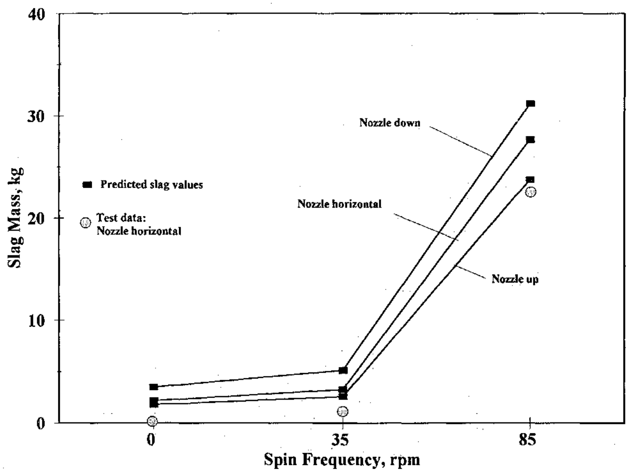

Fig. 2 In-chamber-accumulated-slag mass at the end of burn vs spinning frequency: data from a STAR 63F Q3 static-test firing with the nozzle horizontal, and potential-flow predictions for alternate nozzle orientations with respect to Earth gravity.

μm, which is close to the value of 150 μm adopted previously[15] for a comparable context, then our predictions lie close to the data, especially the higher-resolution data, throughout the burn (Fig. 1). We also compared our predictions with total-accumulated-slag data obtained for no rotation, and also at two finite rotational rates[2] for 112-s-long test burns of the STAR 63F perigee-kick solid rocket motor, which had a length-to-diameter ratio of roughly unity. Because our particle-trajectory calculations include the gravitational body force as well as the interphase drag, we are able to compare results, computed for various nozzle orientations with respect to gravity, with the test results (executed with the nozzle horizontal). With the mass-mean particle size held at the nominal value ($\sim$115 μm), the comparison (Fig. 2) is again favorable.

We conclude, in agreement with other recent work,[7] that the computational complexity of incorporating a finite vorticity distribution across streamlines to enforce the no-slip boundary condition in an inviscid model is unwarranted for the purpose of estimating slag accumulation in aluminized composite grains in practical solid rocket motors.

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