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Technical Comments

Comment on "Simple Modeling of Particle Trajectories in Solid Rocket Motors"

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THE subject paper[1] presents a method for computing slag capture in a solid rocket motor that decouples the gas-particle interaction. First, a potential flow model is used to compute the gas flowfield; second, a Lagrangian particle tracking scheme computes the trajectories of the condensed phase. Since the slag capture is determined by the particle paths, which in turn depend on the gas-dynamic drag, an accurate flowfield is a necessary component of the method. This Comment questions the assumption that the gas flow is potential on both theoretical and experimental grounds, and suggests that an inviscid, vortical flow is more appropriate. In addition, an analytical comparison of the potential flow solution with the vortical solution advocated herein for a geometry typical of a solid rocket motor shows that the gas velocities predicted by the two methods may differ by more than an order of magnitude and by nearly 90 deg at selected locations.

Reference 1 is one in a series of three papers by the same group of authors.[2,3] All use the same assumptions to address slag capture issues. Interestingly, the authors of Refs. 1–3 are aware of and cite Culick's[4] 1966 paper, which showed that in order to properly satisfy the flow boundary conditions at a solid-propellant burning surface, a vortical solution was required. Culick compares the analytical solutions for vortical and potential flows in a constant bore radius motor and states that "a better approximation, more consistent with the burning process, should satisfy the condition that the velocity is normal to the surface."

Most of the arguments in favor of using the potential flow model for the gas are contained in the first[1] of the three papers. One of these arguments,[1] that cites Bachelor[5] for support, states that the specification of the vorticity in the solid-motor inviscid flow is arbitrary. A careful reading of Bachelor indicates that he is referring to the two-dimensional equation with vorticity, but without imposition of the boundary conditions, when he states the "vorticity distribution is arbitrary, so far as inviscid-fluid theory is concerned."[5] Furthermore, he notes the constancy of the vorticity along inviscid streamlines, thus limiting any arbitrariness. Hence, the two-dimensional differential equation, plus the two components of the velocity vector specified at the propellant surface, constitute a well-posed, boundary-value problem by implicitly defining the vorticity on each streamline. One way to show that this problem is well posed is to consider the incompressible, inviscid, two-dimensional, primitive-variable formulation of the fluid equations and use the methods of Courant and Hilbert[6] to find the characteristics. A single real characteristic is found, which coincides with the streamlines, in addition to the two imaginary characteristics associated with the simpler, potential flow. Thus, an additional boundary condition above and beyond the usual potential flow condition is required at boundary locations where streamlines enter the solution region. In the present case, this condition is, as suggested by Culick,[4] that the tangential velocity at the propellant surface is zero.

A subsequent argument in favor of the use of potential flow invokes Goldstein's[7] work on boundary-layer theory. It is stated that "Conventionally for inviscid flow, a vortex sheet is introduced at the boundary to enforce zero slip."[1] One must be careful in applying boundary-layer theory to the flow inside rocket motors. In the case of a boundary layer on a solid, impermeable surface, a vortex sheet indeed can be introduced to enforce zero slip at the wall. With the viscosity neglected, this vorticity does not enter the flowfield, since the convective velocity is tangent to the vortex sheet and the normal diffusion has been neglected. This inviscid solution is irrotational only outside the vortex sheet and satisfies the wall boundary conditions on both components of the velocity. Also, it is a first approximation to the complete solution if the viscosity is small and the boundary layer thin. When the viscosity is small but non-negligible, the vortex sheet diffuses and convects to form the usual boundary layer. However, in the case of interest here, the flow boundary is not a solid surface but is a propellant burning surface. Hence, a solution with a vortex sheet at the boundary is not a valid first approximation to the flow, because the flow is not parallel to the vortex sheet. The vorticity is immediately convected into the flowfield. Therefore, a potential flow with a vortex sheet at an inflow boundary (a propellant burn surface) is not a valid approximation for even the inviscid flow.

Theoretical considerations aside, there is experimental validation of the Culick[4] vortical formulation by Dunlap and his coworkers[8,9] that has been overlooked by the authors of the subject papers. In the earlier paper,[8] it is pointed out that the vortical flow solution advocated herein is an inviscid solution of the incompressible flow equations that satisfies the viscous boundary conditions. Therefore, it is concluded that the solution should agree with real flows if the Reynolds number is large. Laminar flow data are presented for a simulated constant radius motor in the Reynolds number range 3×10^3 to 24×10^3 that are in excellent agreement with Culick's[4] model. The second[9] of these papers provides further experimental validation of the theoretical model; the laminar flow results of the previous paper[8] in which the viscous effects are negligible are confirmed. For turbulent flow, it is found that the pressure force still is larger than the shear force, but only by a factor varying between 2–10. The result is that the measured turbulent velocity profiles agree with the theoretical inviscid solutions in the core flow region, but there is some deviation near the wall where the shear is highest. Figure 21 of Ref. 9 shows that the measured centerline flow velocity drops about 5% below the linear, vortical, inviscid prediction due to turbulence; the predicted, potential-flow, centerline

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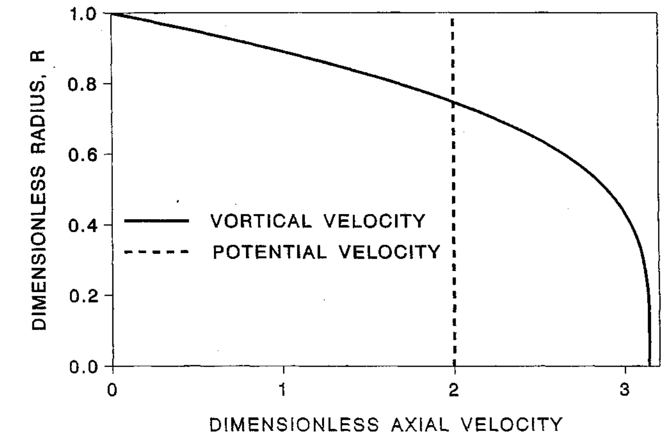

Fig. 1 Axial velocity profiles.

velocities are 36%[4] below the vortical predictions. Therefore, the vortical flow model is a superior flow model in both the laminar and turbulent regimes.

Culick's[4] paper gives both the rotational and irrotational solutions for the flow inside a cylindrical port motor with a constant-radius grain. Solid rocket motors that fly in the atmosphere have large length-to-diameter ratios (10–30 are typical values), and for purposes of illustration, the vortical and potential axial velocities are compared, herein, for a 10-to-1 geometry. As will be obvious, differences between the two solutions increase as the length-to-diameter ratio increases.

The dimensionless axial velocity profiles may be obtained easily[4] as

$$u/(V_b X) = 2 \tag{1}$$

for the potential flow, and

$$u/(V_b X) = \pi \cos(\pi R^2/2) \tag{2}$$

for the rotational flow. Where V_b is the flow velocity at the propellant burn surface, r_0 is the radius of the burn surface, and X and R are the dimensionless axial and radial coordinates normalized with r_0. The quantity X is measured from the motor head end. These velocity profiles are plotted in Fig. 1 and show significant differences both near the wall and on the axis. The largest discrepancy in the flow velocity occurs at $X = 20$, $R = 1$. At this point, the vortical solution gives a radial velocity of V_b and an axial velocity of 0; the corresponding values for the potential solution are V_b and $40V_b$. Thus, the potential flow has a velocity magnitude error of about 40 and a flow angle error of about 90 deg. Since the drag of large droplets varies as the velocity squared, the slag particle drag at that point is in error by a factor of about 1600 in magnitude and 90 deg in direction.

Various investigators[1–3,10] are using potential flow models to compute the gas flow in a two-step slag model. An additional problem with the potential flow approach is that users of these models usually assume some arbitrary vortex shape in the re-entrant region. It has been reported by Smith-Kent and Perkins[10] that this assumed shape has a large effect on the predicted slag accumulation. There is also evidence that, at least in some motors, this separation region is not nearly as large as is frequently assumed. Misterek et al.[11] computed the viscous and inviscid flow for the PAM motor and found the separated flow region to be so small that the viscous solution differed insignificantly from the inviscid solution. It is believed that this occurred in the motor considered in Ref. 11 because the flow in the re-entrant region is radially inward and therefore always experiences a favorable pressure gradient.

This Comment has shown that there is no theoretical basis for the potential flow approach. Comparisons with experimental data have shown good agreement with the analytical, vortical solution published by Culick.[4] Finally, some simple graphical and numerical comparisons of Culick's analytical, vortical, and potential solutions demonstrate that use of the latter can be expected to produce large errors in the slag particle trajectories and, hence, the predicted slag capture.

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Reply by the Authors to J. W. Murdock

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OUR objective in Refs. 1–3 is to estimate the slag accumulation (in the combustion chamber of a solid rocket motor with a metallized composite grain) during the burn, and especially to estimate the total slag accumulation at the end of the burn. The practical motivation is to suggest altered grain composition and/or initial grain configuration, to reduce the performance-degrading accumulation. We seek to replicate the flowfield in the motor during the burn only to the accuracy necessary for a practically useful estimation of the slag retention. Because we seek to avoid any nonessential

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