JOURNAL OF PROPULSION AND POWER
Vol. 10, No. 3, May–June 1994

Axial Compressor Performance During Surge

I. J. Day*
University of Cambridge, Cambridge CB3 ODY, England, United Kingdom

There is limited information available in the literature about flow conditions in axial compressors during surge. This article presents detailed measurements from a low-speed test rig instrumented to pick up details of axial and circumferential flow disturbances. The results show that surge is initiated by rotating stall, and that the ensuing surge cycle is a sequence of well ordered cause-and-effect events. The differences in cycle behavior between "classic surge" and "deep surge" are investigated, and it is shown that the shape of the compressor characteristic determines which of these will occur. From the results it is also concluded that some important factors, such as overall pressure rise and size of hysteresis loop, have not received sufficient attention in existing techniques for predicting the rotating stall/surge boundary. In line with these findings an Appendix by E. M. Greitzer presents a more general version of the "B Parameter," which takes into account the influence of compressor design variables on the stalling behavior of the compressor.

Introduction

THE upper boundary for stable compressor operation is traditionally termed the "surge line." In reality, the compressor may not surge at all at this point, but go into rotating stall instead. The two phenomena, rotating stall and surge, are very different. Rotating stall is a circumferential disturbance of the flow in the region of the compressor blading, while surge is a disturbance which affects flow conditions throughout the entire compression system. While both types of instability have damaging consequences, and are avoided at all costs, it is important to be able to predict which will occur in a particular situation. It is also useful, from a structural design point of view, to know what actually happens in the compressor during an instability event. The purpose of this article is to look at the details of the flow during surge and to examine some of the hitherto neglected factors in predicting whether a compressor will stall or surge.

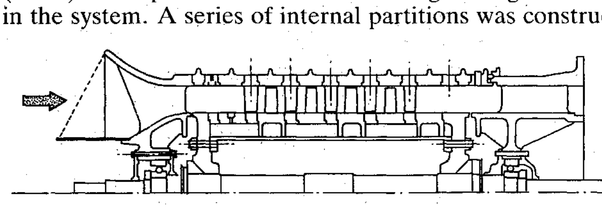
Detailed experimental studies of flow conditions during *rotating stall* have been undertaken by Day and Cumpsty,[1] Lavrich,[2] and others. However, similar detailed work on *surge* is hard to find. Greitzer[3] presented some measurements, but, being primarily interested in system stability, he did not dwell on asymmetric effects during the surging process. Small and Lewis[4] used two circumferentially spaced transducers to pick out circumferential effects, but limited results were reported. Ishii and Kashiwabara[5] also presented some experimental measurements, but lack of detail led them to conclude that surge induces stall, instead of the other way around, as is shown by the present measurements.

A gap thus exists in the literature concerning details of the flow during surge. This aricle presents measurements from a specially instrumented experimental facility giving pressure and velocity measurements at various circumferential and axial positions throughout the system. The results explain what happens in the compressor during surge and suggest ways in which current stability predictions can be improved. It should be noted that the compressor used in these tests is a low-speed machine and, therefore, cannot simulate some of the compressible effects suggested by Mazzawy[6] and Cargill and Freeman.[7] However, it has recently been found that events in an eight-stage aeroengine compressor are very similar to those reported here, even at speeds approaching full speed,[8] and therefore this work is thought to be both realistic and practical.

Experimental Facility

The experimental facility consists of four basic components: 1) the compressor, 2) a parallel annulus duct, 3) a large plenum chamber, and 4) a throttle valve. The compressor, shown in Fig. 1, has four identical stages preceded by a lightly loaded set of inlet guide vanes. The hub/casing radius ratio is 0.75, and the speed of rotation is 3000 rpm, giving a maximum Reynolds number of 1.7×10^5, based on chord. The blading is of modern "controlled diffusion" design and is representative of current high-pressure (HP) compressor practice. A list of basic design details are also given in Fig. 1.

Two different lengths of parallel annulus ducting were used to connect the compressor to the plenum so that the total compressor plus duct length was either 2.0 or 3.5 m. The inner tube forming the duct hub was sealed at both ends to isolate this volume from the rest of the system. A "20 foot" shipping container was used as a plenum with cross-bracings to ensure that wall deflections were kept to a minimum, i.e., less than 0.3% volume at maximum pressure rise. This is small (<4%) in comparison with the mass storage changes occurring in the system. A series of internal partitions was constructed



Mid-Height Blading Details and Other Parameters

	Rotor		Stator
Solidity	1.47		1.56
Aspect Ratio	1.75		1.75
Chord (mm)	35.5		36.0
Stagger (deg.)	44.2		23.2
Camber (deg.)	20.0		40.6
No. of aerofoils	58		60
No. of IGVs		60	
Axial Spacing (mm)		13.0	
Tip diameter (mm)		508	
Hub/Tip ratio		0.75	
Speed of Rot. (rpm)		3000	
Reynolds Number		1.7×10^5	

Fig. 1 Cross-sectional view of the C106 compressor with table of basic design details.

Received Aug. 13, 1991; revision received July 15, 1993; accepted for publication Nov. 5, 1993.

*Senior Research Associate and Fellow, Whittle Laboratory.

so that tests could be run at seven different volumes, giving a range of B parameters from 1.5 to 0.3. A variable speed throttle valve, of negligible duct length, was used to control the flow leaving the plenum.

It will be shown below that stall cell development plays a crucial role in the surging process. To trace the development of the stall cells, six hot-wires equally spaced about the circumference of the compressor were used. Previous work by Day[9] has shown that the stalling process in this machine always starts at the front of the compressor, and therefore, the hot-wires were positioned just upstream of the first rotor. The wires could of course be moved to other stations along the compressor and the annular ducting. Fast response pressure transducers were used to measure pressure fluctuations in the compressor and plenum, while positive flow velocities were obtained from the inlet bell-mouth calibration.

Surge Cycle Measurements

It was found by Greitzer[3] that there are two types of surge cycles. These are termed classic surge and deep surge and their differences are illustrated in Fig. 2. In the case of classic surge, the compressor operating point describes a roughly oval cycle centered about the throttle line. The pressure and velocity fluctuations are smaller than in the case of deep surge and the mass flow rate remains positive. For deep surge the mass flow fluctuates more extensively, becoming negative for part of the cycle. In reversed flow the instantaneous operating point moves down a line known as the negative flow characteristic. This line defines the resistance which the rotating blades offer to flow in the reversed direction. The line is thought to be unique to a particular compressor and is roughly parabolic in shape, as was shown by Gamache and Greitzer.[10] For flow in the positive direction, the operating point closely follows the steady-state characteristic.

The most common mode of instability encountered in this work was deep surge, and therefore the experimental results are presented in a way which emphasizes the various phases of this cycle. However, some details of classic surge were obtained and these are included where appropriate.

Overall Cycle

Figure 3 shows the first, and part of the second, of a series of repeating surge cycles. The upper trace is from a hot-wire positioned upstream of the first rotor and records the axial velocity, while the lower trace shows the corresponding pressure variation in the plenum chamber. For the first 25 revolutions the compressor operates in a stable manner near the peak of the pressure rise characteristic. During this time the throttle is moved very slowly towards the surge point, the flow rate decreasing about 1% for every 50 rotor revolutions. At revolution 25 a burst of rotating stall is observed and the pressure in the plenum begins to fall. The rotating stall is short lived (about six rotor revolutions) and is then replaced by axisymmetric reversed flow, as detailed in a later section. About 40 revolutions after the start of the cycle the compressor unstalls and the plenum pressure begins to rise. If the throttle is not moved from the setting at which the cycle began, a second cycle will be initiated when the compressor stalls again at peak pressure rise.

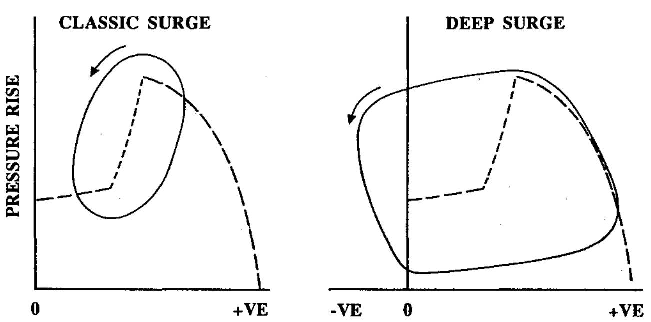


Fig. 2 Difference between classic surge and deep surge.

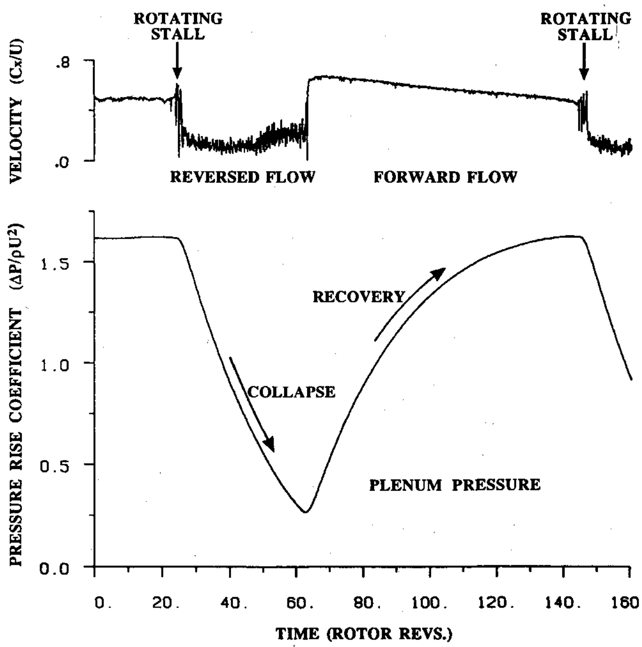


Fig. 3 Axial velocity ahead of first rotor and plenum pressure during first and part of second, surge cycle.

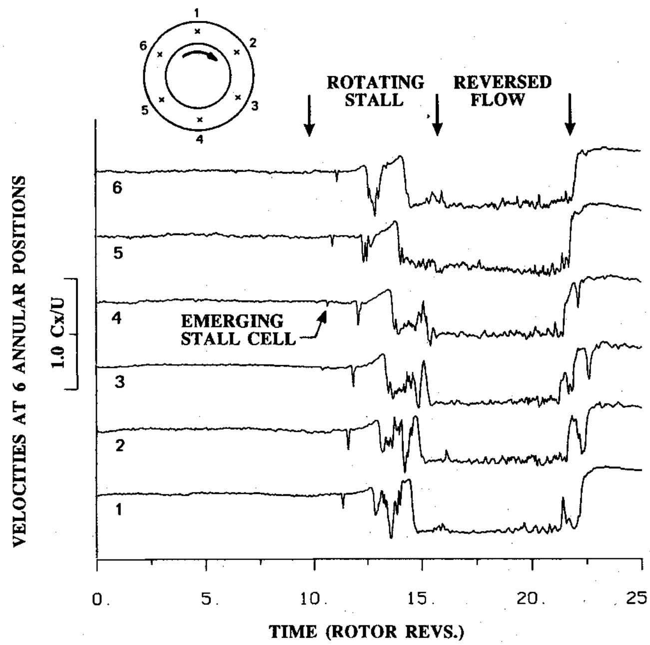


Fig. 4 Hot-wire measurements showing rotating stall at start of deep surge cycle.

In these terms, surge in an *axial* compressor can be regarded as a sequence of cause and effect events triggered by the onset of rotating stall. The analysis of surge as a one-dimensional disturbance beginning from perturbations of linear origin does not, therefore, describe the actual process for cases where rotating stall causes dramatic changes in the operating condition of the compressor. In such cases surge is more of a collapse-and-recovery process than a true dynamic instability. In *centrifugal* compressors, on the other hand, rotating stall often has little effect on compressor pressure rise, and here, one-dimensional linear instability analysis is appropriate.

Rotating Stall at the Start of the Cycle

The stalling process which initiates the surge cycle in the current compressor is shown in greater detail in Fig. 4. Six

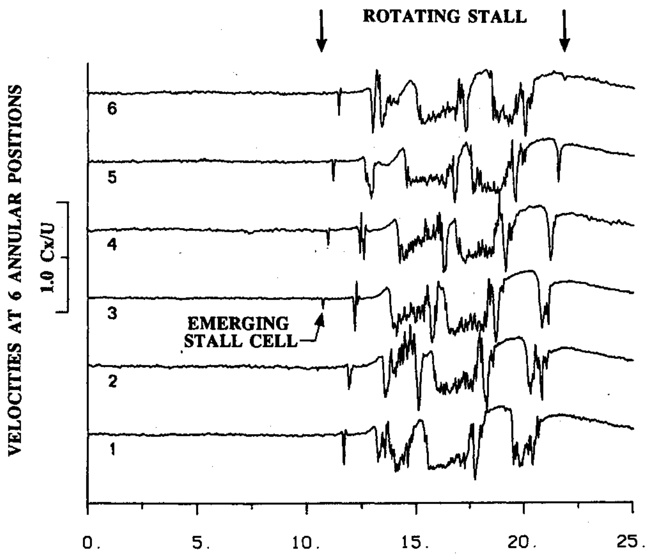


Fig. 5 Hot-wire measurements showing rotating stall during classic surge cycle.

hot-wires are used ahead of the first rotor, equally spaced around the circumference. Prior to the emergence of the stall cell, the flow in the compressor is steady. Once the cell forms, it rotates around the annulus, growing rapidly in size, until it occupies the entire annulus and the flow is fully reversed. This process takes about 4 or 5 rotor revolutions. (Later in the cycle when the compressor unstalls, the process is reversed, i.e., a "cell" of clean flow forms and grows to unstall the compressor. For reasons not yet explained the unstalling process requires less than half the time for the stalling process.)

The above description of surge as beginning with the onset of rotating stall is described in the context of a deep surge cycle. The onset of classic surge occurs in the same way, as shown in Fig. 5, but in this case the flow never fully reverses and the stall cell retains its identity throughout the low flow part of the cycle. In Fig. 5 the stall cell executes five complete revolutions before the compressor unstalls again. [Note that in order to obtain this information on classic surge, it was necessary to operate the compressor at low speed with a modified exit duct. (The modifications to the exit duct consisted of using six flat plates to divide the length of the duct into parallel segments. The effect of the plates was to reduce the size of the hysteresis loop of the compressor.) Under normal operating conditions the natural instability was deep surge.] Further details of stall inception in this compressor are given by Day.[9]

Reversed Flow Measurement

With the six hot-wires positioned upstream of the first rotor, additional information can be obtained if each hot-wire is turned so that the sensing element is parallel to the compressor axis. Measurements in this position show that during reversed flow, the circumferential component of velocity is steady and slightly greater than blade speed. Traverses also show that the flow is uniform over the entire span of the blades. It can thus be concluded that during deep surge the negative part of the cycle is made up of flow which is axisymmetric and steady.

To determine the flow rate during the negative phase of the cycle, it is necessary to take measurements downstream of the compressor where the flow is known to be predominantly axial. The circumferentially averaged result from six hot wires positioned at midspan near the plenum end of the outlet duct is shown in Fig. 6. Here, the reversed flow velocity appears as a positive quantity because of the bidirectional nature of a hot-wire probe. (The fact that the flow is actually

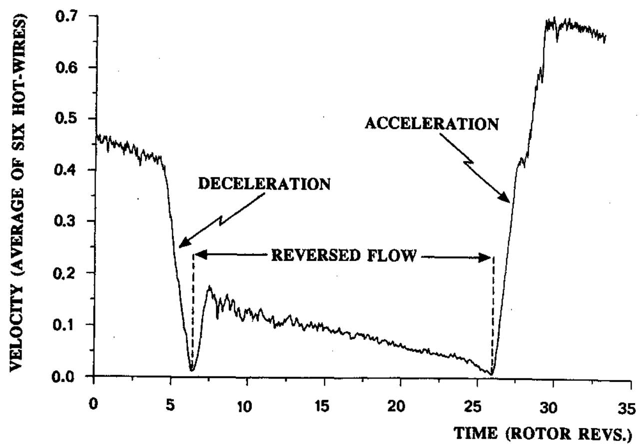


Fig. 6 Average velocity from six hot-wires downstream of compressor exit showing details of flow reversal during deep surge cycle.

reversed was not in any doubt after smoke tests were performed while watching from inside the plenum chamber!)

Figure 6 shows that during the onset and development of the rotating stall the flow in the compressor duct decelerates through zero to negative values. Once the flow is fully reversed there follows a period during which the rate of flow decreases steadily as the plenum chamber empties and the driving pressure decreases, i.e., as the compressor operating point moves down the reversed flow characteristic. (This occurs between revolutions 7–27 in Fig. 6.) When the plenum pressure has fallen to near atmospheric, and the reversed velocity has dropped to near zero, the compressor unstalls and the flow accelerates again.

A typical deep surge cycle can thus be divided into four parts: 1) deceleration, 2) negative flow, 3) acceleration, and 4) normal flow. With the system in the configuration corresponding to Fig. 6, the proportion of time required for each part is roughly 4, 28, 3, and 65%. These proportions change quantitatively with the size of the plenum (as the plenum gets smaller the long-time scale proportions shrink), but the relative magnitudes are qualitatively similar and illustrate how abruptly the flow switches from one direction to the other. The reversed flow phase is understandably shorter than the normal flow phase because in reversed flow the plenum is emptying through the throttle *and* the compressor.

Pressure Forces During Surge

The deceleration and acceleration of the flow in the compressor duct during surge is driven by pressure differences at either end. These can be demonstrated by comparing the static pressure just downstream of the compressor with the pressure in the plenum. Figure 7 covers a complete surge cycle and shows two areas of significant pressure difference: one decelerating the flow when the compressor stalls, and the other accelerating it again when the compressor unstalls. The pressure traces have been labeled to show that the pressure differences are in the correct direction to decelerate and then accelerate the flow. (Note that the pressure at the exit of the compressor is modulated slightly by the close proximity of the stall cell.) An integration of "pressure times flow area" for the shaded regions in Fig. 7 correlates very well with the "mass times acceleration" of the fluid in the duct.

During the surge cycle another period of flow deceleration occurs when the plenum chamber is being pressurized again, i.e., as the flow rate decreases and the compressor operating point rises up the steady-state characteristic. This deceleration is relatively slow and is roughly constant during the "normal flow" phase of the cycle, as can be seen from the hot-wire measurement in Fig. 3, revolutions 65–145. To achieve this deceleration, the pressure in the plenum chamber must be slightly higher than at the compressor exit, as is confirmed in Fig. 7 between revolutions 25–45.

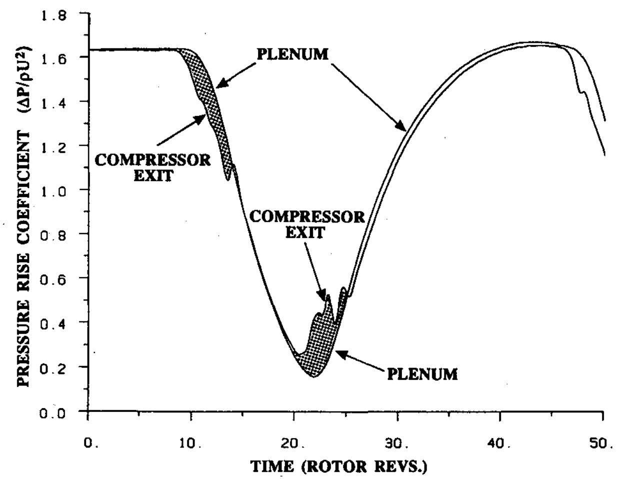


Fig. 7 Measurements at compressor exit and plenum showing the regions of pressure difference which decelerate and accelerate the flow in the ducting.

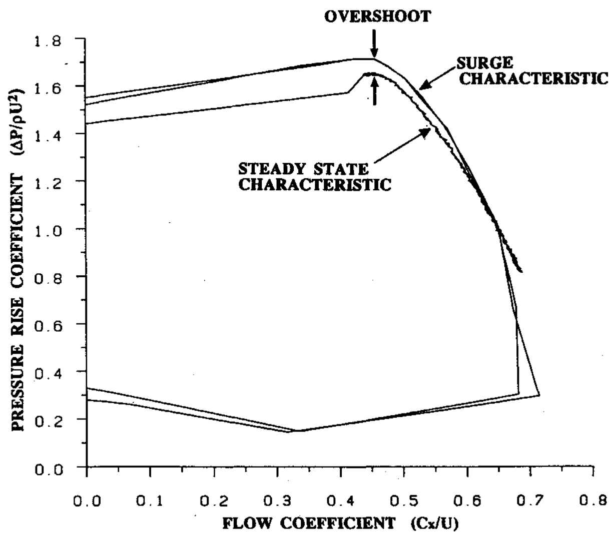


Fig. 8 Comparison of steady-state and dynamic compressor performance.

There is another way in which this decelerating pressure can be observed. Figure 8 shows an example of an on-line recording of a surge event where the operating point moves slowly up the steady-state characteristic before executing two fast surge cycles. (Only the positive flow portion of the cycle is shown here.) The increased plenum pressure needed during the normal flow phase of the cycle to decelerate the flow can be seen as an overshoot at the peak of the characteristic. (The overshoot is most noticeable at this position where the lines are horizontal, but it is actually present further down the characteristic as well.) Because the overshoot is due to the dynamic deceleration of the flow during surge, it becomes more noticeable as the plenum chamber is reduced in size. A smaller plenum leads to a shorter cycle duration, and therefore, to quicker deceleration and more overshoot.

Stall vs Surge

If the size of the plenum chamber is progressively reduced, a point is eventually reached at which the compressor will no longer surge, but will go into rotating stall—all other factors remaining equal. The volume at which the changeover occurs is termed the "critical volume." Figure 9 illustrates the behavior of the system at a volume just less than the critical value. As before, a hot wire is used just ahead of the first rotor, along with a pressure measurement in the plenum wall. In this configuration the compressor stalls and never recovers, but because the volume is so close to the critical value there

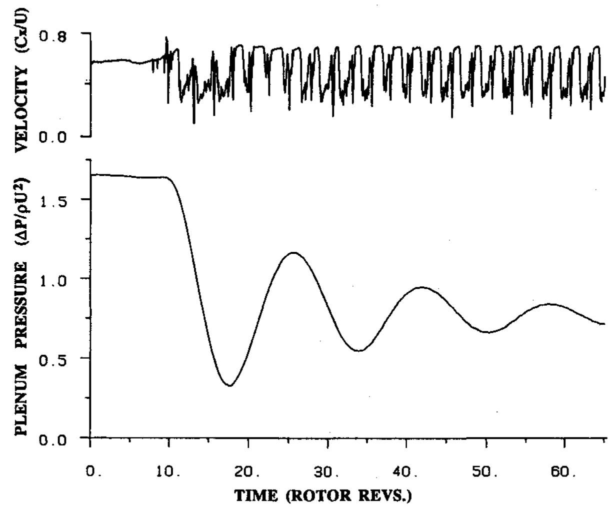


Fig. 9 Velocity and plenum pressure measurements for a stalling compressor with a plenum volume just less than critical.

is a period of decaying oscillation before the compressor settles down to stable operation on the stalled part of the characteristic.

The transient pressure oscillations shown in Fig. 9 are accompanied by a fluctuation of the mass flow through the compressor. As the velocity inside and outside the stall cell remains almost constant, this mass flow fluctuation is achieved by an increase or decrease in the width of the stall cell, i.e., by a change in the respective areas of stalled and unstalled flow. This oscillation in cell size is visible in Fig. 9, and is in overall accord with the parallel compressor model of rotating stall.[11]

Cycle Trajectories and the Negative Flow Characteristic

The mass flow rate during the negative phase of the surge cycle is not easily measured. The inlet bell-mouth calibration is invalid for reversed flow, and hot-wire measurements are unreliable because of temporal drift and the effects of temperature rise in the reversing flow. An alternative method of obtaining the complete surge characteristic is to calculate the flow through the compressor based on the rate of change of pressure in the plenum. If the fluid in the plenum is treated isentropically there is a direct relationship between the rate at which the pressure in the plenum changes and the rate at which mass enters or leaves this volume. The mass leaving through the throttle is dependent on the instantaneous plenum pressure, and therefore, the difference between the rate of mass stored and the rate of mass leaving gives the instantaneous mass flow rate through the compressor, either positive or negative. A complete characteristic for the compressor can thus be built up using only the plenum pressure and throttle characteristic as input. This approach is described in greater detail by Greitzer.[3]

Using this mass balance technique, the surge characteristics for three different plenum volumes are shown in Fig. 10. The steady-state characteristic is included for reference. Also shown is a line drawn through the experimental data to represent the reversed flow characteristic. This line is parabolic in form, with a positive intercept at zero flow. In reversed flow the compressor can therefore be regarded as a throttling device with a positive pressure bias.

Each surge trajectory in Fig. 10 follows the reversed flow characteristic for a short time, the extent of the overlap decreasing as the volume of the plenum is reduced. This can be explained as follows: the rotating stall event which initiates the surge cycle starts at the peak of the characteristic and grows to cover the entire annulus by the time the reversed flow characteristic is intercepted. In this brief period, the pressure in the plenum falls as flow escapes through the throttle. Because the mass stored in the plenum is proportional to

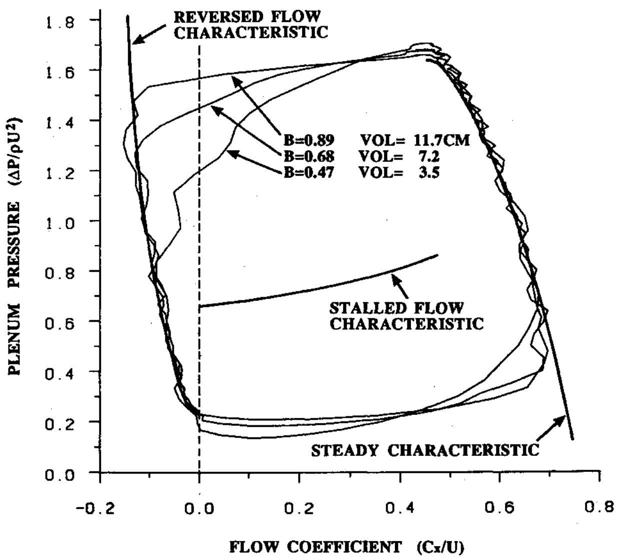


Fig. 10 Performance characteristics during surge for three different plenum volumes.

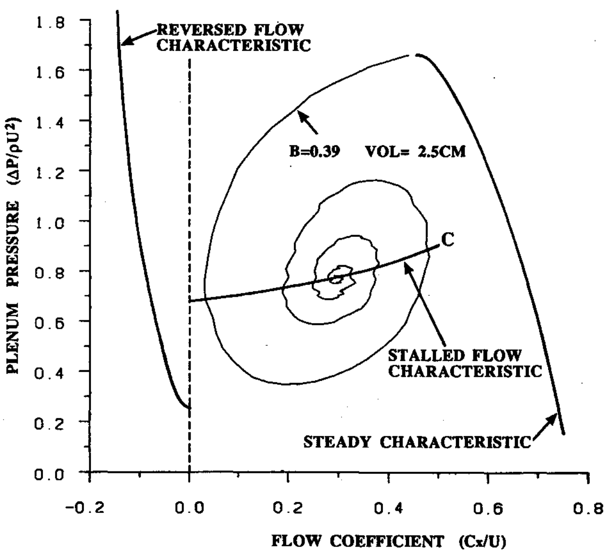


Fig. 11 Trajectory of compressor operating point going into rotating stall with a plenum volume just less than critical.

the volume, the pressure falls further in the case of the smaller plenum, and hence the reversed flow characteristic is intercepted at a lower level. Once on the reversed flow characteristic, the compressor operating point follows the curve in a quasisteady manner down towards the point at which transition to unstalled flow occurs.

If the plenum is made still smaller than in the examples above, a point will be reached where the compressor will exhibit rotating stall rather than surge, as was illustrated in Fig. 9. This point occurs when the plenum becomes so small that after stall onset the plenum empties so quickly that there is insufficient back pressure to completely reverse the flow in the duct. The dynamic characteristic for a compressor with a plenum volume just less than critical, is shown in Fig. 11. After wide excursions in mass flow, the operating point settles down on the fully stalled characteristic. Even though the mass flow excursions are large, the operating point does not intersect the reverse flow characteristic and deep surge does therefore not occur. Likewise, on the return swing, the operating point does not pass the end of the stalled characteristic, point C, and therefore, recovery is also ruled out, i.e., classic surge does not occur. The only alternative is an inward spiral towards rotating stall.

Helmholtz Resonance and Surge Frequency

During the experiments, small pressure fluctuations were sometimes detected in the plenum as the compressor operating point approached the peak of the characteristic. These fluctuations were found to agree closely in frequency with the Helmholtz resonance for the system. A comparison of the calculated and measured frequencies is shown in Fig. 12. The agreement is good considering that the pressure fluctuations were of small amplitude and not easy to measure. Also included in Fig. 12 are the measured surge frequencies for all the volumes tested. Throughout the range, the surge frequency is much lower than the Helmholtz frequency, thus supporting the idea that surge in a multistage axial compressor is more akin to a relaxation oscillation than a Helmholtz resonator-type phenomenon.

In the past, Helmholtz resonator-type perturbations have been assumed to be the linear precursors of surge. The experimental data suggests, however, that Helmholtz fluctuations are only of secondary importance. In all observed cases it is the onset of rotating stall which triggers surge, not the growth of the Helmholtz resonance. It is true that in many instances the point of stall inception coincides with the low velocity point in the Helmholtz wave, but this is not always the case. The Helmholtz waves are of small amplitude and stall cells may form at any time. Hence, the surge cycle need not develop in phase with the Helmholtz wave. An example

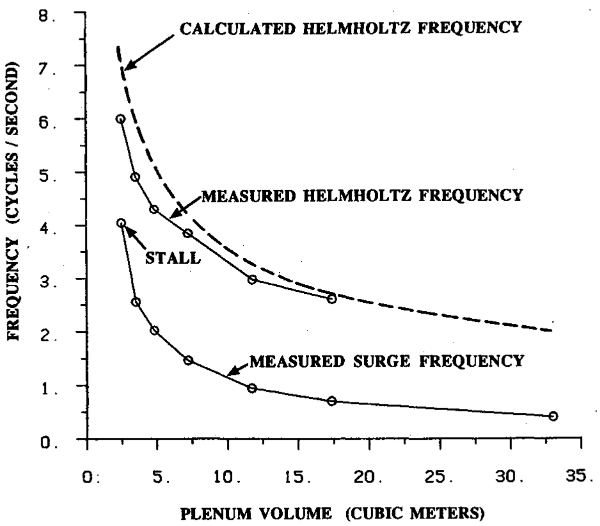


Fig. 12 Comparison of system frequencies.

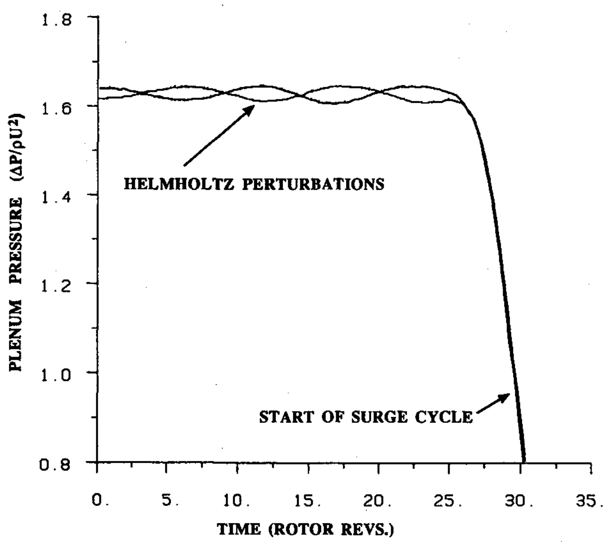


Fig. 13 Plenum pressure measurements for two separate surging events showing that the surge cycle does not develop in phase with the Helmholtz perturbation.

of this is given in Fig. 13 where the plenum pressures for two separated surging events are aligned to show that the start of the surge cycle does not always occur in fixed relation to the Helmholtz wave. In Fig. 13, the stalling of the compressor actually interrupts the Helmholtz resonance. The Helmholtz wave is not a necessary requirement for stall, and therefore, it is also not necessary for surge. It is simply an additional disturbance of limited influence.

In terms of active control, these observations suggest that a one-dimensional control system, which attempts to suppress surge by smoothing out the Helmholtz resonance, will not be very effective in a compressor where rotating stall is the triggering mechanism for surge. The localized velocity reduction associated with the trough of the Helmholtz wave is undoubtedly conducive to stall cell formation, but a velocity reduction of the same magnitude would occur anyway if the flow were throttled just a little more. Damping of the Helmholtz resonance will thus add little to the compressor surge margin. (It should be noted that in many centrifugal compressors, and in some axial fans, the characteristic remains continuous at stall, rather than discontinuous or steeply sloped, and the surge cycle may therefore have true one-dimensional origins.) Stabilization with a one-dimensional actuator will therefore be possible.[12,13]

Discussion

The influence of physical parameters such as plenum volume and duct length on the dynamic behavior of a compression system are well understood. The part played by the design parameters of the compressor itself are less well understood, and it is this topic which is considered in this section.

Greitzer[3] undertook a study of system stability using a one-dimensional lumped parameter model. The flow in the compressor, throttle, and the associated ducting were considered incompressible, whereas the fluid in the plenum was regarded as compressible and isentropic. From the equations of motion for this system a nondimensional grouping of variables was identified which could be used to indicate if a particular compression system would stall or surge. This grouping of variables is now referred to as the "Greitzer B Parameter" and is defined as follows:

$$B = (U/2a)\sqrt{(V/AL)}$$

where U is the midheight blade speed, a the speed of sound, V the plenum volume, and A and L the effective area and length of the compressor and ducting, respectively. For any combination of U, V, A, and L which gives a value of B greater than a certain critical value, Bcrit, a given compression system will exhibit surge, whereas a value of B less than Bcrit points to the likelihood of rotating stall. The effectiveness of this parameter grouping has been demonstrated many times over the past 15 yr, and particularly by the current series of tests.

For the designer of a new installation an estimate of B is easily obtained, however, it will be of little value to him in assessing the chances of stall or surge unless he has prior knowledge of the value of Bcrit. Bcrit is *not* a universal number and is different for each compressor. This was pointed out by Greitzer[14] and is also discussed by McCaughan,[15] but it is worth emphasizing again because of an observed tendency for people to assume that Bcrit is invariant. In the current experiments the critical value of B was found to be about 0.4, whereas Greitzer[3] found a value of 0.8. A difference of this order means that for two compressors of essentially similar details, the volume of the plenum at which the stall/surge changeover will occur would be different by a factor of 4, a large difference from the designer's point of view!

The expression for B contains specific information about the compressor installation, i.e., plenum volume, cross section, and duct length, but it contains only the term U, rotational speed, to specify the performance details of the compressor. In the case of the C106 and the Greitzer experiments, the physical geometry of the two test rigs was similar, and the compressors were designed for the same speed, yet widely different values of Bcrit were obtained. Obviously, more than just the speed of the compressor is responsible for setting up the dynamic behavior of the system. It is proposed here that the missing elements affecting the dynamics of the system are contained in the details of the compressor design. The performance characteristics for the C106 and Greitzer compressors are shown on the same scale in Fig. 14. It can be seen that in spite of similar rotational speeds, the two compressors are very different in terms of pressure rise and the size of hysteresis loop.

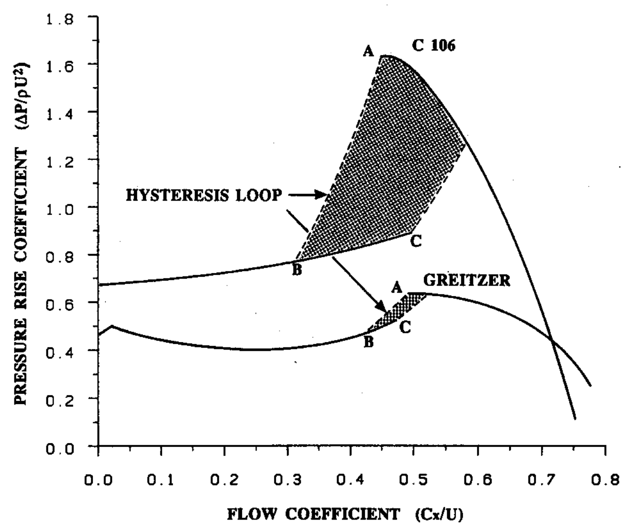


Fig. 14 Comparison of the characteristics for the C106 compressor and the Greitzer compressor.

To examine the effect of the shape of the characteristic on the surging behavior, consider the case where the compressors are operated with negligible plenum volume. In Fig. 14 this would mean that in each case the operating point would move from A to B when stall occurs. For both compressors, B is the steady-state operating point in rotating stall and also the point about which the mass flow will oscillate if the plenum volume is increased. The type of mass flow oscillation considered here is like that in Fig. 11, where the trajectory spirals inwards towards the eventual operating point in rotating stall.

In the case of the Greitzer characteristic, a comparatively small looping oscillation about point B will take the compressor operating point past the end of the hysteresis loop, point C, and back onto the unstalled part of the characteristic. In other words, a small mass flow loop will stall and unstall this compressor in the mode of classic surge. For the C106, on the other hand, a much larger mass flow oscillation would be required to reach the end of the hysteresis loop and unstall the compressor. The oscillation required in this particular case is so large that reversed flow actually occurs before this point is reached, thus causing deep surge. The shape of the Greitzer characteristic therefore favors classic surge, whereas the C106, with its greater gap between point B and the end of the hysteresis loop, C, favors deep surge. A study of the effects of hysteresis was conducted by Greitzer[14] where calculations show that, for the same value of B, an increase in hysteresis leads to a decrease in the likelihood of surge.

The other aspect of compressor design which plays a part in determining the dynamic behavior of the system is the overall pressure rise. In general, the greater the peak pressure rise of the machine, the greater will be the pressure difference between points A and B in Fig. 14. At the instant of stall onset, A can be taken as the pressure in the plenum and B the approximate pressure at compressor exit. The air in the ducting between these two points will be subject to a pressure difference and it is the size of this difference which determines the size of the flow oscillation loop; large oscillations are in general associated with surge and small oscillations with rotating stall. The compressor pressure rise, which affects the distance between points A and B, and hence the size of the flow oscillations, is thus important in deciding between stall and surge.

Given that the size of the hysteresis loop and the overall pressure rise play a part in determining poststability behavior, it would be useful to include these factors in the definition of B to make Bcrit a universal number applicable to all compressors. To this end a dimensionless parameter is defined here which takes into account the compressor design in at least a rudimentary manner. The details of the analysis are given in the Appendix. The overall concept is similar to that described by McCaughan,[15] but makes explicit use of the peak pressure rise and stall mass flow in normalizing the characteristic. If we assume that all compressor characteristics have *roughly* the same overall shape, any particular characteristic can be reduced to a standard format by normalizing with its own peak value of ψ and ϕ, i.e., the pressure rise and flow coefficient at the peak of the characteristic. The relevant dimensionless grouping is thus

$$B' = B\psi_p/\phi_p$$

where subscript p denotes the peak value. For the Greitzer characteristic in Fig. 14 this new parameter gives

$$B'\text{crit} = 0.8 \times 0.64/0.48 = 1.1$$

Recent results from work at United Technologies Research Center on a compressor of intermediate pressure rise also gives a value of B'crit just above unity.[16]

Applying the new parameter to the results from the C106 gives reasonable agreement with the above two compressors, although there is still roughly a 20% discrepancy:

$$B'\text{crit} = 0.37 \times 1.62/0.45 = 1.33$$

The B' parameter includes the effects of pressure rise and axial velocity, *but not of hysteresis*, and it is this factor which is thought to produce the above disparity. In a previous paragraph it was suggested that the size of the hysteresis loop can affect the disposition towards classic surge or deep surge. This disposition depends primarily on the size of the flow oscillation necessary to swing the compressor operating point beyond the end of the hysteresis loop and back onto the unstalled characteristic. The value B'_{crit} for a compressor with a small hysteresis loop going into classic surge will be different, i.e. lower, from that for a compressor with a large hysteresis loop going into deep surge.

The notion of there being two different values of B'_{crit} for classic surge and deep surge is a detail which cannot be included rigorously within a one-dimensional model. Presumably, this aspect could be addressed in a two-dimensional analysis along the lines of that carried out by Moore and Greitzer.[17] For the simple one-dimensional approach presented here, however, the idea does explain why the C106 compressor has a different B'_{crit} value compared with the Greitzer and UTRC compressors, both of which exhibit classic surge. Nonetheless, the new definition of the B parameter is seen as a useful extension of the old one and it is suggested that a compromise value of $B'_{\text{crit}} = 1.1 \sim 1.3$ may prove practical in the future.

Conclusions

1) The experimental results show that for the compressor tested, both classic and deep surge are initiated by rotating stall. In an axial compressor such as this, it is the collapse of the pressure rise associated with the onset of rotating stall which provides the driving force for the flow oscillations associated with surge. (Other measurements from high-speed compressors also confirm this point.[8])

2) Small amplitude axisymmetric velocity fluctuations were observed during testing and were identified as the Helmholtz resonance of the system. These fluctuations were of secondary importance in the surging process because, as described above, rotating stall is a far more vigorous destabilizing force.

3) The surge cycle frequency is determined by the quasi-steady process of "pumping-up" and "blowing-down" the plenum chamber. The surge frequency in both classic and deep surge modes was found to be much lower than the Helmholtz frequency.

4) The compressor used in these tests had a characteristic with a large hysteresis loop and surged in the deep surge mode. The compressor used by Greitzer[3] had a much smaller hysteresis loop and surged in the classic mode. This information, supported by detailed mass flow measurements, shows that the shape of the characteristic and the size of the hysteresis loop are important factors in deciding between surge and stall, and between classic surge and deep surge.

5) Axisymmetric reversed flow occurs during deep surge and measurements show that a unique pressure drop vs mass flow characteristic exists for operation in this mode. The reversed flow characteristic is roughly parabolic in form and has a positive pressure bias at zero flow. During deep surge, operation switches between the unstalled characteristic and the reversed flow characteristic as the plenum fills and empties.

6) The results emphasize that the critical value of the Greitzer B parameter which differentiates between stall and surge in a particular installation is not the same for all compressors. An improved dimensionless grouping, which includes details of the compressor design, rather than just the compressor speed, is presented here. The improved parameter has been assessed in terms of existing data and numerical values have been found to fall in the following range: $B'_{\text{crit}} = 1.1 \sim 1.3$.

Appendix

A derivation is given of a nondimensional grouping that extends the existing B parameter to incorporate, in an approximate manner, the effect of compressor design parameters. The starting point is the equation derived in the Appendix of Ref. 14 for the stability of a compression system.

If the mass flow and plenum pressure perturbations are of the form

$$\delta\phi, \quad \delta\psi \sim e^{st}$$

the equation for s, the nondimensional growth rate, is

$$s^2 + s\left[1\Big/\left(B\frac{\mathrm{d}T}{\mathrm{d}\phi}\right) - B\frac{\mathrm{d}\psi}{\mathrm{d}\phi}\right] + \left\{1 - \left[\left(\frac{\mathrm{d}\psi}{\mathrm{d}\phi}\right)\Big/\left(\frac{\mathrm{d}T}{\mathrm{d}\phi}\right)\right]\right\} = 0 \tag{A1}$$

The quantities appearing in Eq. (A1) are the conventional ones: 1) ψ is the overall (inlet to plenum) pressure rise, normalized by $\frac{1}{2}\rho U^2$, T is the throttle pressure drop, also normalized by $\frac{1}{2}\rho U^2$, and ϕ is the nondimensional mass flow, C_x/U.

The quantity $(\mathrm{d}\psi/\delta\phi)$ is the nondimensional slope of the compressor characteristic and $(\mathrm{d}T/\mathrm{d}\phi)$ is the slope of the throttle characteristic.

The term in square brackets in Eq. (A1) is the most important one in practical circumstances. For *instability* (surge), this term must be negative. Thus

$$B^2\frac{\mathrm{d}\psi}{\delta\phi}\frac{\mathrm{d}T}{\mathrm{d}\phi} > 1 \qquad \text{for instability} \tag{A2}$$

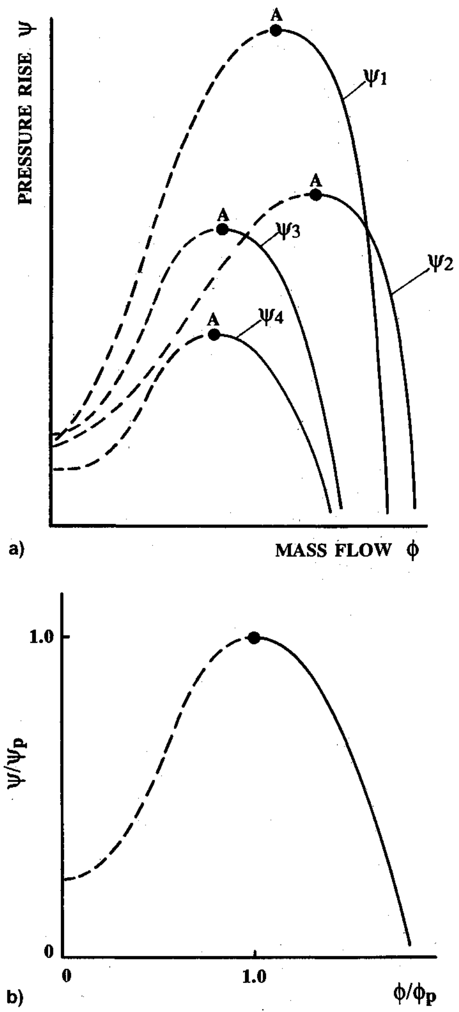


Fig. A1 a) Compressor characteristics from various machines emphasizing a fundamental similarity in the shape of the curves and b) preceding compressor characteristics normalized to a single curve using ψ/ψ_{peak} and ϕ/ϕ_{peak}.

The values of compressor and throttle slope are dependent on design, and for different compressors the critical value of B will also be different. Suppose, however, that for some class of axial compressors all the characteristics can be said to have the same general shape as those in Fig. A1a, even though the peak values of ψ and ϕ (i.e., points A) can differ for different designs. If we normalize ψ by ψ_{peak}, ϕ by ϕ_{peak}, and T by ψ_{peak}, the normalized slopes are given by

$$\frac{d\psi}{d\phi} = \frac{\psi_p}{\phi_p}\frac{d(\psi/\psi_p)}{d(\phi/\phi_p)}$$

$$\frac{dT}{d\phi} = \frac{\psi_p}{\phi_p}\frac{d(T/\psi_p)}{d(\phi/\phi_p)}$$

As indicated in Fig. A1b, normalizing in this manner scales all the compressor characteristics in Fig. A1a to one common curve, with normalized $\psi/\psi_p = 1$, $\phi/\phi_p = 1$ at the peak point.

Substituting in the criterion for instability, Eq. (A2) yields

$$B^2\left(\frac{\psi_p}{\phi_p}\right)^2 \frac{d(\psi/\psi_p)}{d(\phi/\phi_p)}\frac{d(T/\psi_p)}{d(\phi/\phi_p)} > 1 \quad \text{for instability} \quad \text{(A3)}$$

If all compressor characteristics did have similar shapes and slopes, then ψ/ψ_p, T/ψ_p would be "universal," and instability would occur when Eq. (A3) is met. The relevant parameter under these conditions is B' rather than B, where B' is defined as

$$B' = (B\psi_p/\phi_p) \quad \text{(A4)}$$

In summary, for a given B, the higher ψ_p, or the lower ϕ_p, the more likely is surge.

Acknowledgments

I. J. Day is extremely grateful to Rolls-Royce plc. for their financial support of the experiments reported in this article. Significant financial input in the form of a cooperative grant was also provided by the Science and Engineering Research Council. Grateful thanks are extended to colleagues at the Whittle Laboratory: T. Camp, R. Day, J. Longley, and especially N. Cumpsty. E. M. Greitzer was on sabbatical leave from "the other Cambridge" when this article was written and his generous help and input have culminated in this collaborative publication.

References

[1]Day, I. J., and Cumpsty, N. A., "The Measurement and Interpretation of Flow Within Rotating Stall Cells in Axial Compressors," *Journal of Mechanical Engineering Sciences*, Vol. 20, No. 2, 1978, pp. 101–114.

[2]Lavrich, P. L., "Time Resolved Measurements of Rotating Stall in Axial Flow Compressors," Gas Turbine Lab. Rept. 194, Massachusetts Inst. of Technology, Cambridge, MA, 1988.

[3]Greitzer, E. M., "Surge and Rotating Stall in Axial Flow Compressors," *Journal of Engineering for Power*, Vol. 98, Pts. I and II, April 1976, pp. 190–217.

[4]Small, C. J., and Lewis, J. T., "High Speed Compressor Rig as a Stall Recovery Research Tool," AIAA Paper 85-1428, July 1985.

[5]Ishii, H., and Kashiwabara, Y., "Surge and Rotating Stall in Axial Compressors," AIAA Paper 89-2683, 1989.

[6]Mazzawy, R. S., "Surge-Induced Structural Loads in Gas Turbines," *Transactions of the American Society of Mechanical Engineers*, Vol. 102, Jan. 1980, pp. 162–168.

[7]Cargill, A. M., and Freeman C., "High Speed Compressor Surge with Application to Active Control," American Society of Mechanical Engineers Paper 90-GT-354, June 1990.

[8]Day, I. J., and Freeman, C., "The Unstable Behaviour of Low and High Speed Compressors," American Society of Mechanical Engineers Paper 93-GT-26, May 1993.

[9]Day, I. J., "Stall Inception in Axial Flow Compressors," *Journal of Turbomachinery*, Vol. 115, Jan. 1993, pp. 1–9.

[10]Gamache, R. N., and Greitzer, E. M., "Reversed Flow in Multistage Axial Compressors," *Journal of Propulsion and Power*, Vol. 6, No. 4, 1990, pp. 461–473.

[11]Day, I. J., Greitzer, E. M., and Cumpsty, N. A., "Prediction of Compressor Performance in Rotating Stall," *Journal of Engineering for Power*, Vol. 100, Jan. 1978, pp. 1–14.

[12]Ffowcs Williams, J. E., and Huang, X. Y., "Active Stabilisation of Centrifugal Compressor Surge," American Society for Mechanical Engineers Paper 90-GT-123, June 1990.

[13]Pinsley, J. E., Guenette, G. R., Epstein, A. H., and Greitzer, E. M., "Active Stabilization of Centrifugal Compressor Surge," American Society for Mechanical Engineers Paper 90-GT-123, June 1990.

[14]Greitzer, E. M., "The Stability of Pumping Systems" (The 1980 Freeman Scholar Lecture), *Journal of Fluids Engineering*, Vol. 103, June 1981, pp. 193–242.

[15]McCaughan, F. E., "Application of Bifurcation Theory to Axial Flow Compressor Instability," *Journal of Turbomachinery*, Vol. 111, No. 4, 1989, pp. 426–433.

[16]Lavrich, P. L., personal communication, United Technologies Research Center, East Hartford, CT, 1992.

[17]Moore, F. K., and Greitzer, E. M., "A Theory of Post-Stall Transients in Axial Compression Systems, Part I: Development of Equations," *Journal of Engineering for Gas Turbines and Power*, Vol. 108, Jan. 1986, pp. 68–76; also Greitzer, E. M., and Moore, F. K., "A Theory of Post-Stall Transients in Axial Compression Systems, Part II: Applications," *Journal of Engineering for Gas Turbines and Power*, Vol. 108, April 1986, pp. 231–240.