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# Design of a Modified Hypocycloid Engine

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## ABSTRACT

The modified hypocycloid engine incorporates a unique geared drive that imparts straight-line, sinusoidal motion to the one-piece piston and rod assembly. These kinematic characteristics provide a variety of potential benefits not possible with traditional slider-crank kinematics. Perfect engine balance is achieved through the use of two sets of counterweights. The absence of piston side thrust promises reductions in piston assembly friction and piston slap, even with smaller piston skirts. Additional potential benefits include improved combustion characteristics and reduced piston manufacturing costs. Although simpler hypocycloid designs provide the same motion, the modified hypocycloid engine reduces gear and crankshaft loading.

A description and design details of a prototype engine currently under construction are presented. Patented design improvements over previous hypocycloid designs are described. These improvements reduce crankshaft stresses, ensure a compact crankshaft with convenient assembly and disassembly, and control deviations from the desired straight-line piston motion that are caused by gear tooth backlash.

## INTRODUCTION

The form of the slider-crank mechanism used in today's internal combustion engines has remained basically intact since its introduction. While this mechanism has demonstrated its effectiveness for over a hundred years, various shortcomings have led to suggestions of

alternative mechanisms for converting gas combustion pressure to crankshaft torque. Among these alternatives are a variety of sinusoidal engines, so named because of the sinusoidal, straight-line motion which the piston undergoes.

This paper provides a description of advantages that sinusoidal engine kinematics provide when compared to slider-crank kinematics. The basic hypocycloid engine is reviewed as one particular form of sinusoidal engine offering distinct advantages over other sinusoidal engines. A modification to the basic hypocycloid engine is introduced which offers the advantage of reduced component loading. Finally, a prototype design of this modified hypocycloid engine is presented. This design includes patented improvements over previous designs, some of which can be applied to the basic hypocycloid engine as well as the modified hypocycloid engine. A prototype modified hypocycloid engine incorporating these features is currently under construction and will be used to verify the design concept.

## SINUSOIDAL ENGINES

Sinusoidal engines have several potential advantages compared to the conventional slider-crank engine, such as perfect balance, reduced piston assembly friction and reduced piston slap, among others. This section provides a brief outline of some of these advantages.

**PERFECT BALANCE** - The slider-crank mechanism does not produce sinusoidal motion along a straight line for a constant crankshaft speed. The acceleration and resulting inertia forces of the mechanism, parallel and

perpendicular to the cylinder axis, are represented by a series of harmonic terms. As a result, the single-cylinder slider-crank engine cannot be perfectly balanced. Instead, counterweights are generally sized to minimize the combination of horizontal and vertical engine bearing loads (1).<sup>\*</sup> The shaking forces due to remaining unbalance are annoying at best, and destructive to engine components and users at worst. Harkness describes extensive balancer developments undertaken to satisfy customer demands for vibration reduction (2). Ishida, et al., note the vibration associated with known piston and crank mechanisms and discuss the problem of nerve damage to forestry workers from the vibration of conventional slider-crank chain saw engines (3,4).

Sinusoidal piston motion of sinusoidal engines offers the significant advantage of allowing perfect balance when compared to conventional slider-crank engines. Piston and connecting rod motion occurs along a straight line at a single frequency, permitting simple balancing. Ishida and Yamada explain the theory of perfect balance provided by a single-cylinder sinusoidal hypocycloid experimental chain saw engine. This engine is of the basic hypocycloid design to be discussed subsequently. Experimental results are provided showing vibration levels of 1/40th that of a conventional chain saw engine (4,5).

**IMPROVED COMBUSTION CHARACTERISTICS** - Increasing environmental concerns and more stringent emissions standards will require improved engine designs to be developed. Sinusoidal engines offer some inherent potential advantages with respect to emissions. Some of the factors that could lead to a positive effect on emissions are the reduced vibration levels and the uniform piston/cylinder clearance around the periphery of the piston made possible by these engines, discussed in more detail subsequently. Low vibration levels should help maintain proper engine adjustments which affect emissions, and uniform piston/cylinder clearance should result in less oil consumption.

Another important aspect when considering the potential for reduced emissions is the amount of unburned hydrocarbons in the exhaust. Experimental and analytical work done

at General Motors Research Laboratory by Reitz and Kuo has clearly demonstrated that the unburned fuel is, in large part, due to the incomplete combustion of fuel in the crevices near the piston rings (6). This unburned fuel is dependent on piston ring dynamics and geometry, particularly as influenced by ring interaction with the cylinder wall and piston. With elimination of piston side loading, the design of the piston and ring set becomes simplified and offers the potential of reducing the size of or eliminating the piston top-land crevice.

In addition, with sinusoidal piston motion, pistons spend more time in the combustion zone at the initiation of the expansion stroke (7,8,9). For similar ignition timing, this should lead to higher peak pressures and temperatures, possibly permitting more efficient burning and greater energy output for a given fuel, or the use of lower grade fuels.

**REDUCED PISTON ASSEMBLY FRICTION** - The contribution of piston assembly friction to engine mechanical losses is widely documented. Kuhlmann and Jakobs suggest that the piston assembly accounts for approximately 40% of overall engine friction losses (10). Hoshi and Baba conclude through their tests that approximately 50% of friction losses of an internal combustion engine occur in the piston system, of which 70-80% are due to piston rings (11). Parker, et al., state that the piston skirt accounts for 10-20% of mechanical losses, which agrees well with Hoshi's conclusions (12). Oh, Li and Goenka note that most analytical studies on piston friction focus on ring lubrication, then develop an elasto-hydrodynamic model of piston skirts which demonstrates the value of careful skirt design in reducing frictional losses (13). Through studies based on their piston skirt model, Oh, et al., predict reductions in piston skirt frictional losses of 19.7%. Assuming that the skirt accounts for 20% of total mechanical losses, this would result in a 3.94% reduction in overall mechanical losses, a significant figure. Li, Rhode, and Ezzat also discuss the significance of the piston skirt in engine frictional losses (14). Their dynamic piston model demonstrates the dependence of piston skirt friction on piston tilt and piston pin offset.

When compared to slider-crank engines, sinusoidal engines offer significant reductions in piston assembly friction. The slider-crank piston skirt acts as a bearing for the piston, minimizing

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<sup>\*</sup>Numbers in parentheses designate references listed at the end of the paper.

piston lateral motion to reduce noise and vibration, as noted by Takiguchi, et al. (15). This lateral motion is caused by forces on the piston perpendicular to the cylinder axis that arise from connecting rod inclination. Unfortunately, reduction of skirt/cylinder clearances to provide better guidance also results in higher skirt frictional forces. The need to provide this guidance is reduced or eliminated with a sinusoidal engine. Forces on the piston are aligned with the cylinder axis so that the tendency for the piston to move transversely and tilt is greatly decreased, reducing the need for a skirt. This, in turn, reduces piston assembly friction.

Another component of piston assembly friction is the piston wrist-pin and wrist-pin bearing. Although only a small portion of total power loss is due to the wrist pin, elimination of any power loss is beneficial, as noted by Burkett (9).

The possibility of eliminating wrist-pin friction exists with sinusoidal engines. The wrist-pin is required in the slider-crank engine to accommodate connecting rod inclination. In the sinusoidal engine, with a connecting rod that travels in a straight line, it may be possible to completely eliminate the wrist-pin and associated frictional losses.

**REDUCED PISTON SLAP** - The wrist-pin for a slider-crank piston is generally offset from the piston centerline to reduce piston slap and resulting noise and liner cavitation problems as the piston moves laterally from one side of the cylinder wall to the other. The significance of these problems is demonstrated by the numerous attempts to analyze and reduce piston slap noise and liner cavitation erosion, as reported in (16) and (17). Straight-line piston motion should eliminate piston slap.

**MANUFACTURING BENEFITS** - Since the sinusoidal engine does not necessarily require a wrist-pin, the piston and rod assembly, or piston-rod, might be machined as one piece, reducing manufacturing complexity and cost. The slider-crank piston is asymmetric due to the wrist-pin boss, causing it to thermally expand unevenly. As a result, it is typically cam ground to an oval shape so it has a circular shape after thermal expansion. This cam grinding is unnecessary with a one-piece, symmetrical, sinusoidal engine piston-rod, further reducing manufacturing costs. If further research indicates that the mobility provided by a joint between the piston and

piston-rod is still necessary for sinusoidal engines, a limited motion ball joint would be adequate, allowing the piston to remain symmetrical.

**LOW HEAT REJECTION ENGINES** - The benefits of sinusoidal engines make them particularly attractive candidates for the low heat rejection engine. The ability to isolate the combustion chamber because of straight-line piston-rod motion is particularly useful. In a conventional engine, the side-to-side oscillation of the connecting rod makes it difficult to effectively seal the crankcase from the engine's upper end. In a sinusoidal engine, the piston rod reciprocates along the cylinder axis without side-to-side oscillation. This straight-line motion facilitates incorporating a seal around the piston rod, thus isolating the crankcase from the upper end of the engine. The ease of sealing the upper end of the engine permits separate lubrication of the upper and lower ends of the engine. This offers the potential of reduced contamination of bearing lubricant and thus longer life and improved lubricant performance. The elimination of piston side loads and the elimination of the wrist-pin significantly changes the lubrication requirements in the upper end of the engine. Mourelatos discusses the possibility of gas lubrication of a piston for low heat rejection engine applications and emphasizes the need for crank/rod mechanisms which minimize piston side thrust (18). Minimization of this side thrust is necessary to avoid hot, high-speed rubs which could damage most material combinations. Because of the higher temperatures associated with the low heat rejection engine, the possibility of utilizing gas bearings in the engine upper end is particularly attractive. Both the ability to isolate the crankcase from the cylinder and the elimination of piston side loads contribute to the feasibility of using gas lubrication or materials which require no lubrication in the upper end of the engine.

An additional concern that is particularly relevant for research engines incorporating ceramics is contamination of main engine bearings due to ceramic break-up. The usual mode of failure of ceramic materials results in fragments which could damage bearings and other components in the engine crankcase. The ability to isolate the crankcase with a piston-rod seal would contain these potentially harmful fragments.

## APPLICATION OF HYPOCYCLOID MOTION TO ENGINE DESIGN - THE BASIC HYPOCYCLOID ENGINE

One means of achieving sinusoidal motion is to utilize a mechanism generating hypocycloid motion. A hypocycloid is the path described by a point on the circumference of a circle which rolls on the inside of a larger, fixed circle. Figure 1 depicts a hypocycloid.

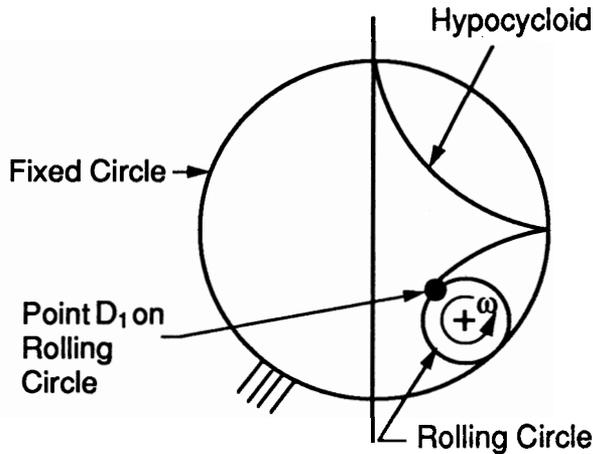


Figure 1 - Hypocycloid Motion

In Figure 1, when the smaller circle has a diameter of 1/2 the larger circle and rolls on the larger circle at constant speed, the resulting hypocycloid is a straight line which has harmonic motion. The application of this concept to engine design results in the hypocycloid engine, a particular type of sinusoidal engine.

Figure 2 portrays a hypocycloid engine, in which the fixed circle is replaced by an internal gear and the rolling circle is replaced by a pinion. The arm is attached to the pinion and the combination is supported at C by a crankpin bearing. As the crankshaft rotates with angular speed  $\omega_c$  in the direction shown, the arm and pinion rotate about the crankpin with angular speed  $2\omega_c$  in the opposite direction. As the crankshaft completes one revolution, point  $D_1$  (and the piston-rod, supported on a bearing at point  $D_1$ ) executes straight-line motion through the crank centerline, O. The motion of point  $D_1$  is simple harmonic for a constant crankshaft speed. The stroke L is equal to the internal gear diameter, which is four times the crankthrow O-C and twice the pinion diameter. Note that point  $D_2$  also performs straight-line, simple harmonic

motion, perpendicular to  $D_1$  and through the crank centerline. Kinematic equations for this hypocycloid motion are found in (9).

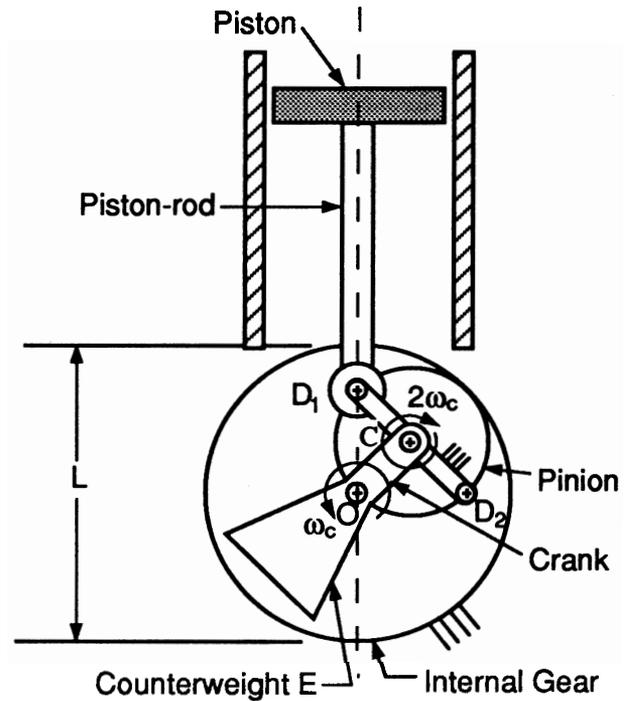


Figure 2 - Hypocycloid Engine

Balance for the single-cylinder hypocycloid engine is achieved through the use of two sets of counterweights, and can be illustrated with the help of Figure 3.

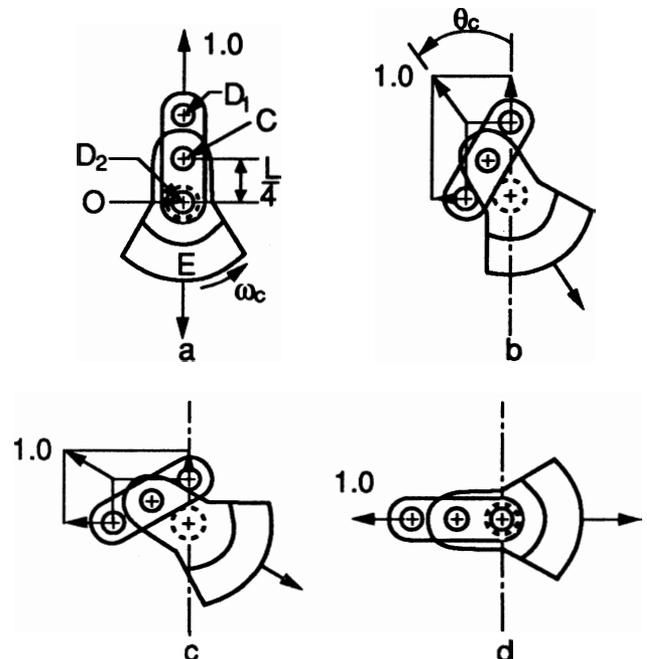


Figure 3 - Hypocycloid Engine Balancing (Beachley, (7))

Parts (a) - (d) of this figure represent 30 degree increments of crankshaft rotation beginning at Top Dead Center in (a) and finishing at 90 degrees rotation in (d). The reciprocating mass of the piston-rod can be thought of as concentrated at point  $D_1$ , which undergoes straight-line simple harmonic motion. The inertia force from this reciprocating mass always passes vertically through point  $D_1$  and varies with the cosine of the crank angle, as shown. A counterweight mass equivalent to the reciprocating mass can be located at point  $D_2$  whose inertia force will be equal to that of the reciprocating mass at  $D_1$ , but at right angles and 90 degrees out of phase. The resultant of these two inertia forces will be a constant force rotating at crankshaft speed and always passing through the crankpin axis  $C$  and crankshaft axis  $O$ , as shown in the figure. This resultant inertia force can be balanced by a second counterweight mass located at  $E$  to achieve perfect balance in the plane shown. In an actual engine the counterweights located at  $D_2$  and  $E$  would be pairs of counterweights in order to balance moments along the crankshaft axis  $O$ . A more detailed discussion of balancing principals for the hypocycloid engine is found in (9).

Beachley and Lenz discuss variations of hypocycloid engines that use the internal gear/pinion pair as described here (7). Among these are the "big bearing" and "built-up shaft" designs. The subject of this paper is the "big bearing" design, which is referred to subsequently as the basic hypocycloid engine. Figure 4 illustrates the basic hypocycloid engine.

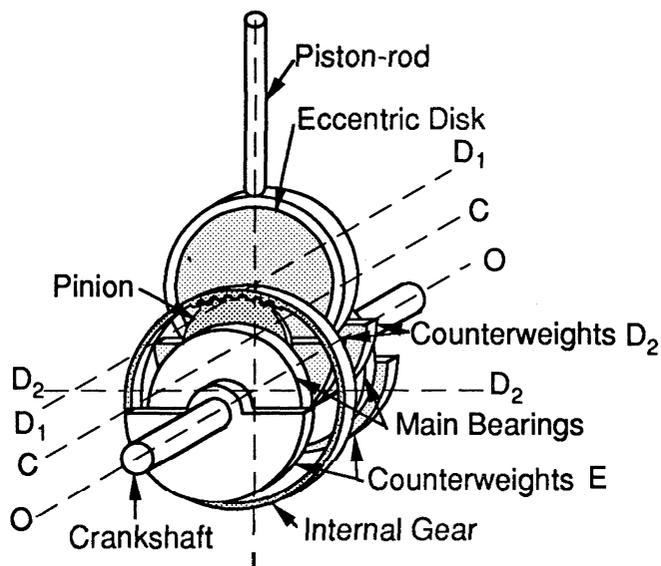


Figure 4 - Basic Hypocycloid Engine

Points  $D_1$ ,  $D_2$ ,  $C$ , and  $O$  from Figure 2 are axes when viewed in Figure 4. The pinion, eccentric disk and two counterweights ( $D_2$ ) seen in this figure are all rigidly connected and rotate together. The eccentric disk, or "big bearing" journal, is used to translate the sinusoidal, reciprocating motion of a point on the pinion pitch diameter to the center of the piston-rod "big bearing". The relatively large size of the bearing is required to allow clearance for the crankpin as the crankshaft rotates through its cycle. This is further illustrated with the help of Figure 5, which shows the relative locations of the crankshaft axis  $O$ , crankpin axis  $C$ , and the "big bearing" center  $D_1$  for 180 degrees of crankshaft rotation.

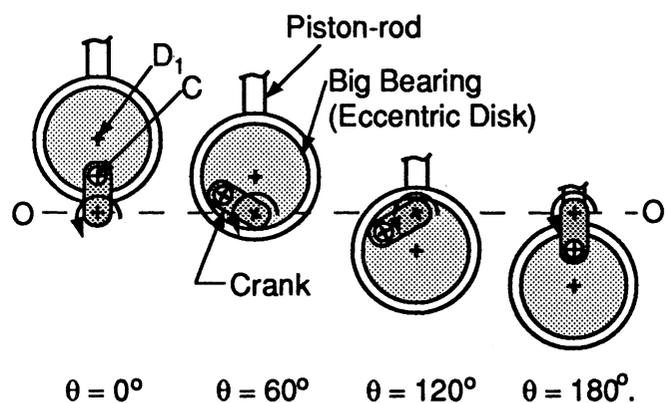


Figure 5 - "Big Bearing"

The use of the basic hypocycloid engine concept is not new. Beachley and Lenz include an extensive list of patents for hypocycloid devices, among which many basic hypocycloid engines are included (7). Burkett tabulates several more recent applications of the basic hypocycloid engine (9). Included in these is an experimental chain saw engine for which vibration data is reported by Ishida, et al. (4,5).

**CHARACTERISTICS OF THE BASIC HYPOCYCLOID ENGINE** - Sinusoidal engines offer the advantages of straight-line sinusoidal piston motion and perfect balance previously discussed. However, the methods for achieving these goals differ between the various engine concepts. The following section discusses several basic hypocycloid engine characteristics.

**Frictional Losses** - Piston-skirt/cylinder friction is largely eliminated in sinusoidal engines by constraining the piston-rod to move in a straight line. In the Scotch-yoke mechanism, this is achieved through the use of piston-

rod guide bearings. The crosshead mechanism achieves the same effect through the use of a crosshead bearing which is attached to the bottom of the piston-rod. These concepts are illustrated in figure 6. In both cases the piston-skirt friction eliminated is replaced by friction from the bearings guiding the piston-rod.

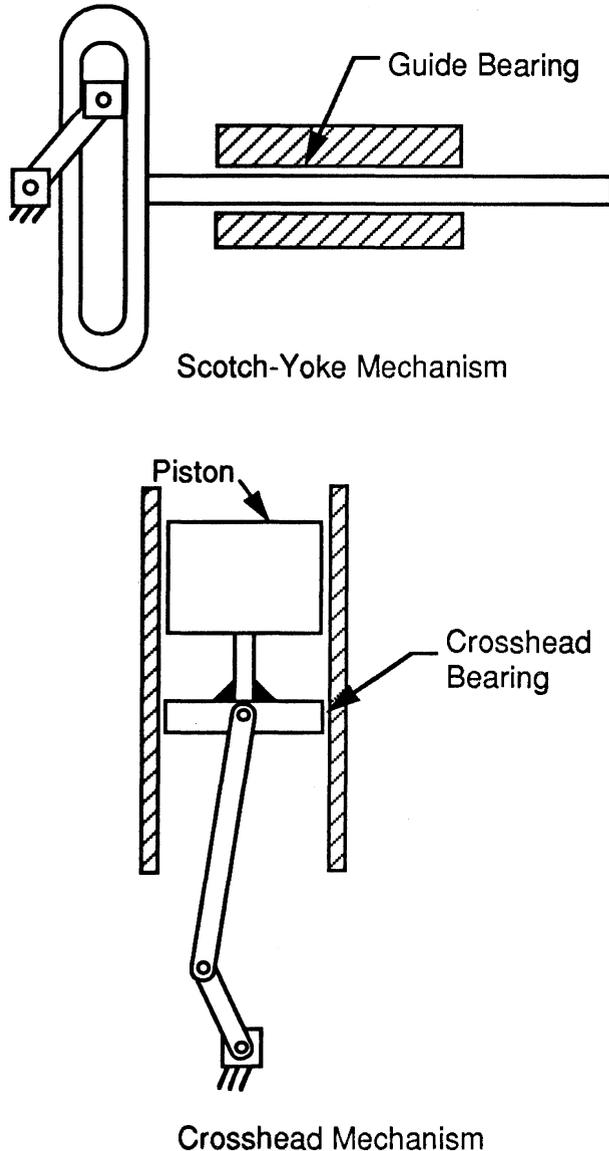


Figure 6 - Sinusoidal Engine Concepts

The basic hypocycloid engine does not use piston-rod guide bearings to provide straight-line piston motion. Instead, the piston-rod is made to move in a straight line by the gearing system. In effect, restraining loads on the crosshead bearing for the crosshead mechanism are replaced by gear tooth loads in the basic hypocycloid engine. Since gearing is typically very efficient, it is anticipated that friction losses due to the basic hypocycloid gear mesh will be

minimal. The gear mesh efficiency for the basic hypocycloid engine can be developed with the help of figure 7, which represents gas loads and gas load reactions on the basic hypocycloid engine pinion at constant crankshaft rotational speed,  $\omega_c$ .

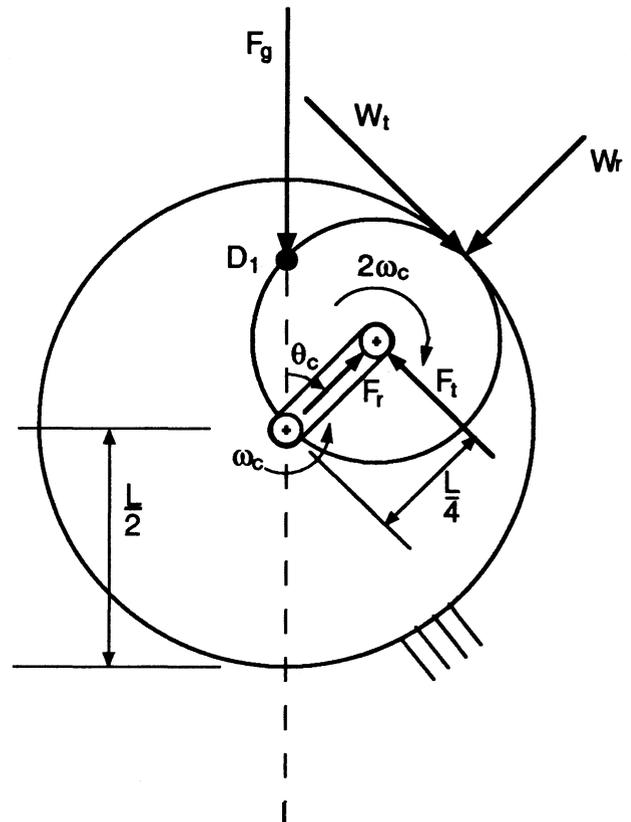


Figure 7 - Basic Hypocycloid Engine Pinion Loads

The resultant inertia load for the piston-rod assembly was previously shown to pass through the crankpin axis radially, as does the inertia load due to pinion rotation about the crankshaft axis. Since these loads pass through the crankpin center, they will not affect subsequent determination of gear tooth loads, and so are omitted from Figure 7. For constant pinion rotational speed  $\omega_c$  about the crankpin, summation of moments leads to

$$W_t = F_g \sin \theta_c \quad (1)$$

where

$$\begin{aligned} W_t &= \text{tangential gear tooth load on the} \\ &\quad \text{pinion,} \\ F_g &= \text{gas force on the piston,} \\ \theta_c &= \text{crankangle.} \end{aligned}$$

Thus, an important result is obtained that the gear loading is only a function of crankangle and the gas force, and is independent of speed. The torque exerted on the crankshaft can be expressed as

$$T = F_t \frac{L}{4} \quad (2)$$

where

$$\begin{aligned} T &= \text{torque on the crankshaft,} \\ F_t &= \text{tangential load on crankpin from} \\ &\quad \text{pinion,} \\ L &= \text{engine stroke.} \end{aligned}$$

Alternatively, the torque on the crankshaft can be expressed as

$$T = W_t \frac{L}{2}, \quad (3)$$

so that combining Eqs (2) and (3),

$$F_t = 2W_t = 2F_g \sin \theta_c. \quad (4)$$

Using Dudley's method for calculating efficiencies in planetary and differential drives (19),

$$\eta = \frac{P_i - \Sigma[\phi P_{eq}]}{P_i}, \quad (5)$$

where

$$\begin{aligned} \eta &= \text{drive efficiency,} \\ P_i &= \text{ideal power transmitted without} \\ &\quad \text{losses,} \\ \phi &= \text{fractional power loss of an individual} \\ &\quad \text{mesh,} \\ P_{eq} &= \text{equivalent power transmitted by an} \\ &\quad \text{individual mesh.} \end{aligned}$$

The equivalent power transmitted by an individual gear mesh can be expressed with the equation

$$P_{eq} = W_t V, \quad (6)$$

where

$$\begin{aligned} W_t &= \text{tangential tooth load transmitted at} \\ &\quad \text{the mesh,} \\ V &= \text{pitch-line velocity of tooth} \\ &\quad \text{engagement at the mesh.} \end{aligned}$$

The pitch-line velocity of tooth engagement for two rotating, fixed center gears is simply calculated as the product of the pitch radius of either gear and its angular velocity. The angular velocity also indicates how many gear teeth have meshed per unit time. For example, for one revolution of the gear, all the gear teeth have meshed once. To calculate  $V$  for cases where the two gears do not necessarily have fixed centers, then, the angular velocity used must indicate how many gear teeth have meshed per unit time. The basic hypocycloid engine has only one mesh, the pinion/ring mesh. From Figure 7 it is seen that the pinion makes two revolutions with respect to the crankpin for every one revolution of the crankshaft. It can also be shown that this means each pinion tooth goes through two meshes for every one revolution of the crankshaft. Therefore, the effective angular velocity of the pinion is twice that of the crankshaft, and

$$V = \frac{L}{4} (2 \omega_c) = (0.5)L \omega_c. \quad (7)$$

$$P_{eq} = W_t V = (0.5)W_t L \omega_c. \quad (8)$$

Using Eq (3), ideal power is found as

$$P_i = (0.5)W_t L \omega_c \quad (9)$$

Therefore,

$$P_{eq} = P_i, \quad (10)$$

and substituting into Eq (5),

$$\eta = \frac{(0.5)W_t L \omega_c - (0.5)\phi W_t L \omega_c}{(0.5)W_t L \omega_c}. \quad (11)$$

A typical power loss for a spur gear mesh might be 0.7% or less ( $\phi = 0.007$ ) (19). Substituting this into Eq (11), the efficiency for the basic hypocycloid engine gearing becomes

$$\eta = 0.993. \quad (12)$$

It is apparent that for the basic hypocycloid engine the efficiency for the gear train is the same as for the individual gear mesh. Therefore, mechanical losses due to the gear mesh in the basic hypocycloid engine should be less than 1.0% of the available mechanical energy.

Several additional sources of frictional loss must be considered for the basic hypocycloid engine. In the slider-crank engine, the connecting rod is supported at its "big-end" by the big-end bearing on the crankpin. In the basic hypocycloid engine, the piston-rod "big-end" is supported on the eccentric disk or "big bearing". This "big bearing" is a source of friction which could be comparable to or greater than the slider-crank big-end bearing friction, depending on bearing type and engine parameters and operating conditions. Additionally, another bearing exists for the basic hypocycloid engine, the crankpin bearing. In figure 4, this is the bearing which supports the pinion, counterweights  $D_2$ , and eccentric disk on the crankpin, indicated by axis C-C. These two bearings must be considered for a complete comparison of friction gains and losses for the basic hypocycloid engine and other engines.

**Absence of Piston-rod Bending** - The piston-rod for the basic hypocycloid engine is caused to move in a straight-line by the gearing, so that no bending loads are imposed on the piston-rod. This is in contrast to the Scotch-yoke engine and other varieties of engines that require piston-rod constraint to create straight-line motion. This is evident when viewing Figure 6, which shows bearing constraint of the Scotch-yoke mechanism that would result in bending of the piston-rod.

**Compact Size** - The internal gearing utilized in the basic hypocycloid engine is attractive when size is to be kept small. Ishida notes this advantage when comparing different sinusoidal engines of hypocycloidal form, some of which incorporate external gearing to achieve linear, harmonic motion (3).

## THE MODIFIED HYPOCYCLOID ENGINE

Although the basic hypocycloid engine has distinct advantages over other sinusoidal engines, it does have one significant shortcoming. The gear load,  $W_g$ , is dependent only on the gas pressure  $F_g$ , and the crankangle,  $\theta_c$ . This results in higher gear tooth loads as cylinder pressures increase. Beachley and Lenz concluded that for typical gas pressure in spark ignition (SI) and combustion ignition (CI) engines, the basic hypocycloid engine should have adequate tooth strength for bore:stroke ratios of approximately 3:4 or less (7). However, for supercharged applications or engines with

greater bore:stroke ratios, tooth stresses may become too high.

A concept which provides for reduced gear loading is the modified hypocycloid engine, which was patented by Huf, et al., in 1980 (20). This concept is similar to the basic hypocycloid engine, except that for a given engine stroke, a pinion is used with a pitch diameter larger than  $1/2$  the engine stroke and an internal ring gear with a pitch diameter larger than the engine stroke. The ring gear is made to rotate at a particular fraction of the crankshaft speed in the opposite direction. This counter rotation is achieved through a sun gear attached rigidly to the crankshaft and planet gears between the sun and the ring gear. This arrangement is represented schematically in Figure 8.

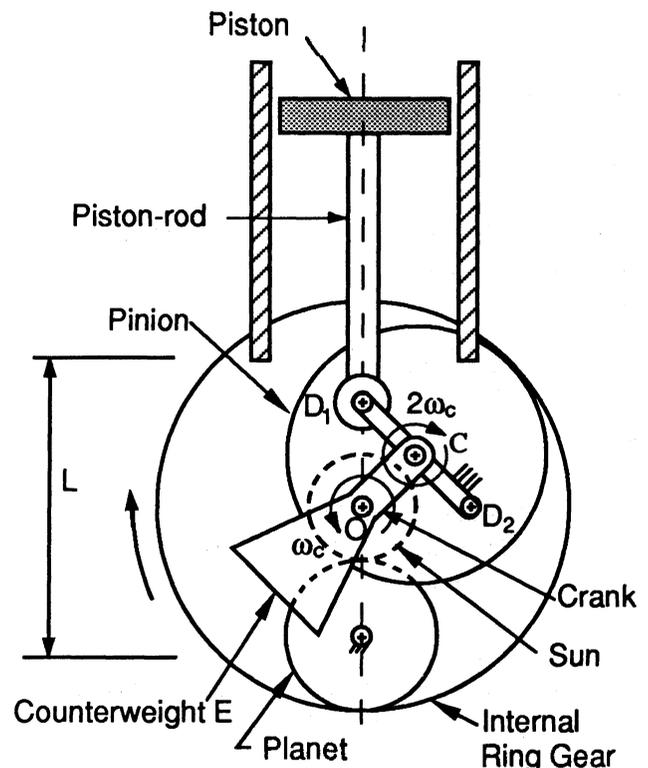


Figure 8 - Modified Hypocycloid Engine

Straight-line motion for the arrangement is maintained by setting the angular velocity of the internal gear relative to the crankshaft rotational speed so that the instant center of rotation of the pinion for the modified design corresponds to the instant center of the pinion for the basic design for all crank angles. This situation is represented in Figure 9.

Figure 10, which represents gas loads and gas load reactions on the modified hypocycloid engine pinion at constant crankshaft rotational

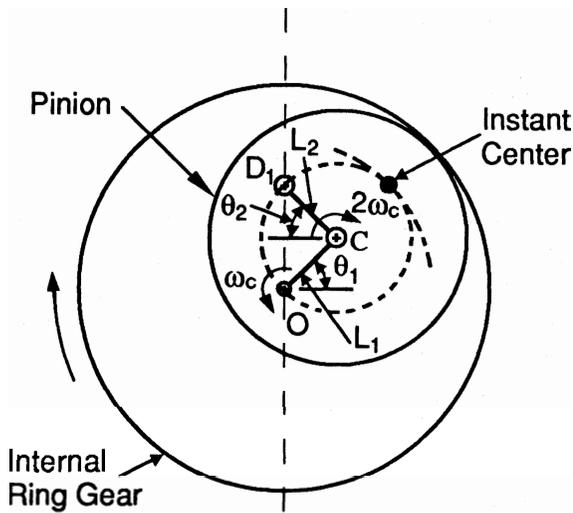


Figure 9 - Modified Hypocycloid Engine Kinematics

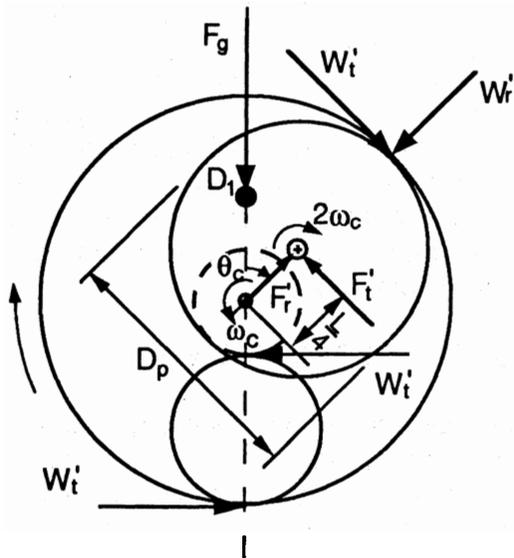


Figure 10 - Modified Hypocycloid Engine Gear Loads

speed,  $\omega_c$ , is useful in determining gear tooth loads. Tangential gear tooth loads for the sun and planet gears are also shown in figure 10, assuming one planet gear. Inertia loads are omitted from the figure since they do not affect the gear tooth loads. For constant pinion rotational speed,  $2\omega_c$ , about the crankpin, summation of moments and Eq (1) lead to

$$W_t' = F_g \sin \theta_c \frac{L}{2D_p} = W_t' \left( \frac{L}{2D_p} \right), \quad (13)$$

where

$D_p$  = pinion pitch diameter,

$W_t'$  = tangential tooth load on modified hypocycloid gearing.

For a given engine stroke,  $L$ , the gear tooth load for the modified hypocycloid engine is reduced from the basic hypocycloid engine by the ratio of the stroke to twice the pinion pitch diameter. This ratio is equal to unity for the basic hypocycloid engine. As the pinion and internal gear diameters increase, the load on the pinion teeth decreases.

Another advantage of the modified hypocycloid engine is the reduction of crankshaft loads caused by the gas pressure,  $F_g$ . For the modified hypocycloid engine, a power split occurs, by which torque is transmitted to the crankshaft through two paths; the first path is through the crankpin and the second through the sun gear. The total torque exerted on the crankshaft can be expressed as

$$T' = T'_{\text{crankpin}} + T'_{\text{sun}} \quad (14)$$

The torque on the crankshaft through the crankpin can be expressed as

$$T'_{\text{crankpin}} = F_t' \frac{L}{4}, \quad (15)$$

where

$F_t'$  = tangential load on modified hypocycloid crankpin from the pinion.

Alternatively, the torque on the crankshaft through the crankpin can be expressed as

$$T'_{\text{crankpin}} = W_t' \left( \frac{L}{4} + \frac{D_p}{2} \right), \quad (16)$$

so that using Eqs (13), (15), and (16),

$$F_t' = W_t' \left( 1 + \frac{2D_p}{L} \right) = W_t' \left( \frac{L}{2D_p} + 1 \right). \quad (17)$$

Comparing this equation to Eq (4), it is seen that the gas loads on the crankshaft are reduced for the modified hypocycloid engine.

The increase in number of gear meshes for the modified hypocycloid engine can also be analyzed from an efficiency standpoint. Referring to figure 10 and rewriting Eq (5),

$$\eta' = \frac{P_i - \Sigma[\phi P_{eq}]}{P_i} \quad (18)$$

For this case, the ideal power transmitted is the sum of power transmitted through the crankpin and through the sun gear to the crankshaft,

$$P_i = T' \omega_c = (T'_{\text{crankpin}} + T'_{\text{sun}}) \omega_c$$

$$= W_t' \left[ \left( \frac{L}{4} + \frac{D_p}{2} \right) + \frac{D_{\text{sun}}}{2} \right] \omega_c. \quad (19)$$

It can be shown that a requirement of straight-line motion for the modified hypocycloid engine is that

$$D_{\text{sun}} = D_p - \frac{L}{2}. \quad (20)$$

This results in an expression for the total power

$$P_i = W_t' D_p \omega_c. \quad (21)$$

Expressions for equivalent power for the meshes can be developed by first rewriting Eq (6) as follows:

$$P_{\text{eq}} = W_t' V. \quad (22)$$

For the sun/planet meshes,  $V$  is simply the product of the sun gear radius and its angular velocity since both gears have fixed centers.

$$(P_{\text{eq}})_{\text{sun/planet}} = W_t' \left( \frac{D_{\text{sun}}}{2} \right) \omega_c$$

$$= W_t' \left[ \frac{D_p}{2} - \frac{L}{4} \right] \omega_c. \quad (23)$$

Similarly, for the planet/ring gear meshes,

$$(P_{\text{eq}})_{\text{planet/ring}} = W_t' \left[ \frac{D_p}{2} - \frac{L}{4} \right] \omega_c. \quad (24)$$

For the pinion/ring mesh, the pinion gear revolves twice for every revolution of the crankshaft and every tooth meshes twice. Therefore, the effective angular velocity of the pinion is twice that of the crankshaft and,

$$(P_{\text{eq}})_{\text{pinion/ring}} = W_t' \left( \frac{D_p}{2} \right) 2\omega_c = W_t' D_p \omega_c. \quad (25)$$

As an example, let the pinion diameter be twice that for the basic hypocycloid engine,

$$D_p = L \quad (26)$$

Then,

$$P_i = W_t' L \omega_c. \quad (27)$$

$$(P_{\text{eq}})_{\text{sun/planet}} = W_t' \left( \frac{L}{4} \right) \omega_c \quad (28)$$

$$(P_{\text{eq}})_{\text{planet/ring}} = W_t' \left( \frac{L}{4} \right) \omega_c \quad (29)$$

$$(P_{\text{eq}})_{\text{pinion/ring}} = W_t' L \omega_c \quad (30)$$

Assuming the same power loss for each mesh as was used for the basic hypocycloid efficiency calculations,  $\phi = 0.007$ , the expression for efficiency of the modified hypocycloid gear set, Eq (18), then becomes

$$\eta' = \frac{W_t' L \omega_c - (0.007)(1.5)W_t' L \omega_c}{W_t' L \omega_c}, \quad (31)$$

$$= 0.990. \quad (32)$$

This efficiency is comparable to that in the basic hypocycloid engine, where the efficiency was estimated to be 0.993. It should be noted that these calculations assume the gearing efficiency is independent of load.

Additional sources of frictional losses must be considered for the modified hypocycloid engine. These include planet gear bearings and the internal gear bearing. The possibility exists of using anti-friction bearings for these locations, particularly for the relatively large diameter internal gear.

## DESIGN OF A PROTOTYPE MHE

The benefits of the modified hypocycloid engine in reducing gear tooth loading and crankpin loading make prototype verification desirable. An engine was designed using the modified hypocycloid mechanism. This engine uses parts from an existing Honda SL90 motorcycle engine. The Honda engine is a single-cylinder, 4-stroke, air-cooled, overhead-cam, spark ignition engine. Values of several engine parameters are shown in Table 1.

Table 1 - Modified Hypocycloid Engine Parameters ( Burkett, (9) )

	Honda SL90E	Modified Hypocycloid
Bore (mm)	50.0	50.0
Stroke (mm)	45.5	47.6
Displacement (liters)	0.090	0.094
Comp. Ratio	8.2:1	8.5:1
Main Bearings	Ball	Journal

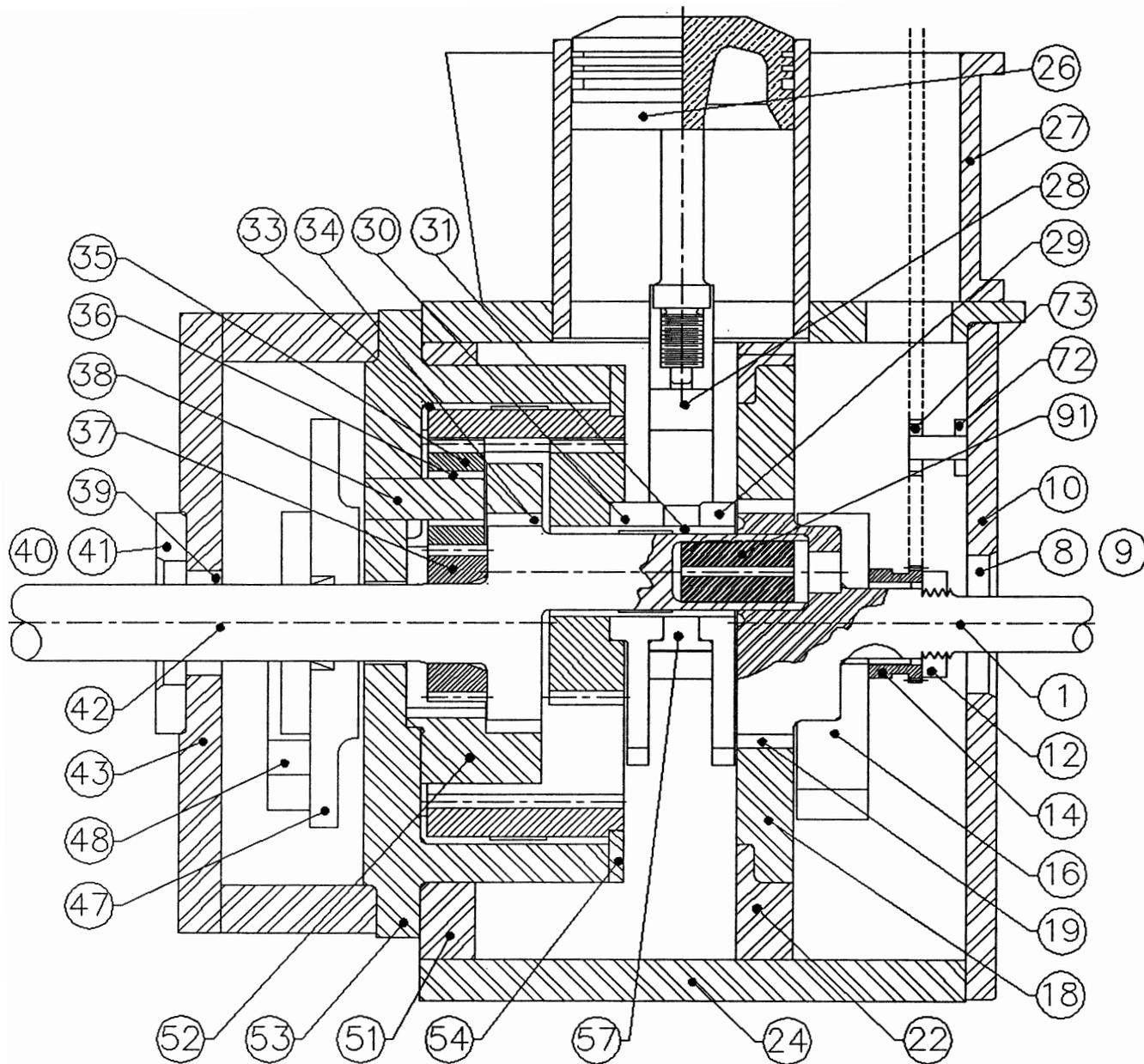


Figure 11 - Modified Hypocycloid Engine Assembly

Figures 8 and 9 show that the pinion gear rotates about the crankpin at twice the angular speed as that of the crankshaft rotation. This relationship is maintained by the particular combination of sun, planet, internal, and pinion gears, which also determines the stroke of the engine. The selected gear set resulted in a stroke slightly greater than for the original slider-crank engine, as noted in Table 1.

This particular Honda engine was chosen for several reasons. The size of the engine is relatively small, which simplifies construction

and reduces associated costs. The overhead-cam feature is also advantageous for conversion to the modified hypocycloid engine. The motion of the counterweights located at point  $D_2$  in Figure 2 makes it somewhat difficult to locate a camshaft in the crankcase. Use of an overhead-cam arrangement eliminates this difficulty.

Figure 11 is an assembly drawing of the modified hypocycloid engine. The following section provides a description of part relationships by major assembly.

**CRANKCASE ASSEMBLY** - The *crankcase* {24} and main crankcase assembly components are constructed from mild steel. The *bearing no. 3 support plate* {18} is bolted to the *cam sprocket shaft bulkhead* {22} and acts as a support for *bearing no. 3* {19}. The *flywheel cover* {43} and *ring gear support* {53} are bolted to the *output shaft bulkhead* {51}. The flywheel cover supports the *no. 1 bearing* {39} and includes an *oil seal* {40,41}. The ring gear support holds the *ring gear bearing* {33}, which is restrained axially by a *retaining ring* {54}. The *bearing no. 2 housing* {52}, which contains the *bearing no. 2* {34}, is bolted to the ring gear support. The 3 hardened steel *planet gear shafts* {38}, which support the *planet gears* {35} and *planet gear bearings* {36}, are press fit into the ring gear support. The bolt holes which connect the ring gear support with the crankcase are oversize, allowing rotation of the ring gear support (ie., planet carrier) with respect to the crankshaft to provide for backlash adjustment. An *oil seal* {8,9} is provided in the *cam sprocket shaft cover* {10}, which is bolted to the crankcase. The *idler bases* {72} are bolted to the cam sprocket shaft cover and support *idler sprockets* {73}, which are used to take up timing chain slack and to adjust valve timing. The *cylinder head* {not shown} and *cylinder block* {27} are attached to the crankcase by four bolts.

**CRANKSHAFT ASSEMBLY** - The *output shaft* {42} is constructed of 4140 steel. The crankweb is made circular so that it can serve as the no. 2 bearing journal and the crankpin provides a surface for the interference fit of the *cam sprocket shaft* {1} with the output shaft. A *crankpin taper plug* {91} is pressed into the crankpin taper hole to expand the crankpin against the cam sprocket shaft. The *sun gear* {37} is shrink fit onto the output shaft against the the web front surface. The *flywheel* {47}, constructed of 1045 steel, is attached to the output shaft with a commercially available locking assembly and the *output shaft counterweight* {48}, also constructed of 1045 steel, is bolted to the flywheel. All outer surfaces of the output shaft are ground.

The *cam sprocket shaft* {1} is constructed of 4140 steel. The cylindrical web of the shaft acts as the journal for the no. 3 bearing. A hole bored in the web receives the crankpin. A woodruff key in the shaft restrains the *cam sprocket shaft counterweight* {16} and *timing sprocket* {14} from rotation. Threads on the front

shaft allow the *cam sprocket shaft nut* {12} to axially restrain the counterweight and sprocket.

**PINION ASSEMBLY** - The pinion assembly is supported by the *crankpin bearing* {31}, which is press fit into the assembly and rides on the crankpin. Pinion and other gear information is found in Table 2.

Table 2 - Gear Data (Burkett, (9))

Name	No. of teeth	Width (mm)	Pieces
Pinion	36	16.8	1
Ring	51	42.7	1
Sun	21	13.5	1
Planet	15	12.7	3

Pressure angle: 20 degrees

Material: 4140 steel

AGMA Class: 9

The pinion is attached with socket head machine screws to the *inner counterweight no. 1* {30}, *eccentric disk* {57} and *inner counterweight no. 2* {29}, all of which are fabricated of 1045 steel. An alignment hole and pin are provided to guarantee correct relative angular alignment of the pinion assembly components. The *piston-rod big end* {28}, is fabricated of aluminum and serves as the "big bearing" for the eccentric disk. The one-piece *piston-rod* {26} is constructed of aluminum and threads into the piston-rod big end. It contains an integral piston, which is shorter than the stock Honda piston. The shorter piston is made possible by the hypocycloid motion, which eliminates the need for a piston-skirt to guide the piston and carry side loads. A commercial thread-locking compound is used to secure the piston-rod to the big end. This is possible since the big-end is located away from the high combustion chamber temperatures. A stock Honda SL90E cast iron piston ring pack is used.

**CYLINDER BLOCK ASSEMBLY** - The cylinder block assembly is the stock Honda SL90E cast-iron block {27}. The bottom end of the cylinder is cut off since a shorter cylinder is possible with a shorter piston.

**CYLINDER HEAD ASSEMBLY** - The head assembly, not shown, is the stock Honda SL90E cast aluminum head.

**GENERAL ASSEMBLY NOTES** - All bearings used are cast bronze journal bearings. The lubrication system consists of an externally

powered oil pump which supplies pressurized oil to the bearings and cooling oil sprays to the piston and gear set. The lubrication system is discussed in detail in (8) and (21). A standard 6 volt or 12 volt battery will be used in conjunction with the Honda breaker points and ignition coil to provide spark ignition for the engine.

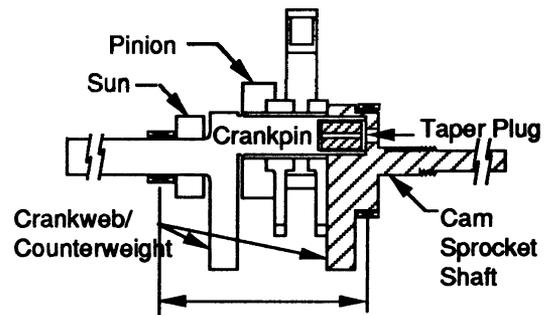
### INNOVATIVE DESIGN CHARACTERISTICS OF THE PROTOTYPE MODIFIED HYPOCYCLOID ENGINE

The distinct structure of the modified hypocycloid engine leads to challenging design opportunities. Accordingly, the focus of research to date has been the development of effective component and assembly designs. This section presents an overview of patented design solutions and an identification of areas for further development (22).

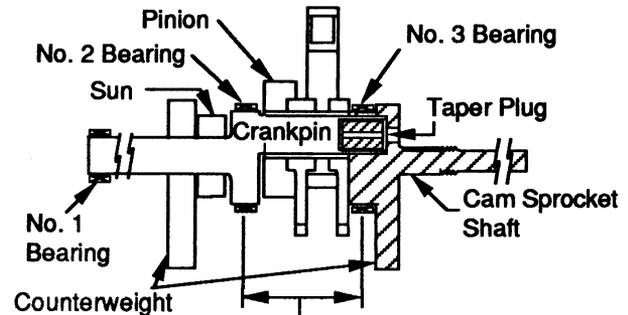
#### CYLINDRICAL CRANKWEB/BEARINGS -

One of the primary differences in the modified hypocycloid engine and slider-crank crankshaft design for a one cylinder engine is the increase in required crankshaft length. The typical slider-crank crankshaft requires axial space between main bearings for the connecting rod big end and two crankweb/counterweights. For the modified hypocycloid engine, the span increases by the widths of two additional counterweights, a pinion gear, and a sun gear if main bearings are located outside the main counterweights, as shown in Figure 12. The increase in bending moments from gas and inertia loads at this greater span would require a significant increase in crankshaft diameter. Design efforts were directed towards changing this configuration.

The resulting crankshaft design is shown in Figure 12. Typically, crankwebs provide the function of counterweights and unions between the main bearing journals and crankpins. By making the crankwebs cylindrical, we have made them main bearing journals, as seen in the figure. Main counterweights are then placed beyond the no. 2 bearing and sun gear. Inertial forces from the cam sprocket shaft counterweight are effectively supported by the no. 3 bearing, since there is little overhang between the two. Because the output shaft counterweight is located relatively far from the no. 2 bearing, an additional bearing, the no. 1 bearing, is added to support its inertia forces.



Bearing Span for Modified Hypocycloid Engine Without Cylindrical Crankwebs



Bearing Span for Modified Hypocycloid Engine With Cylindrical Crankwebs

Figure 12 - Modified Hypocycloid Engine Bearing Span

**CRANKSHAFT TAPER PIN PRESS FIT -** Another design challenge for the modified hypocycloid engine is the installation of the pinion assembly on the crankshaft. In many slider-crank engines, the connecting rod big end is split; this facilitates the removal of the connecting rod and piston for routine inspection and maintenance. An analogous arrangement for the modified hypocycloid engine would involve the separation of pinion, inner counterweights, piston-rod big end and eccentric disk, as shown in Figure 13. This method of construction was considered, but rejected for several reasons. Although split gears have been satisfactorily fabricated and successfully run, it was felt that for a prototype engine with large tooth loads a one-piece pinion would be more reliable, as well as simpler to manufacture. A split arrangement would also require a method to hold the halves together. This would necessitate bulkier design for all assembly components to allow for bolts or other fastening devices.

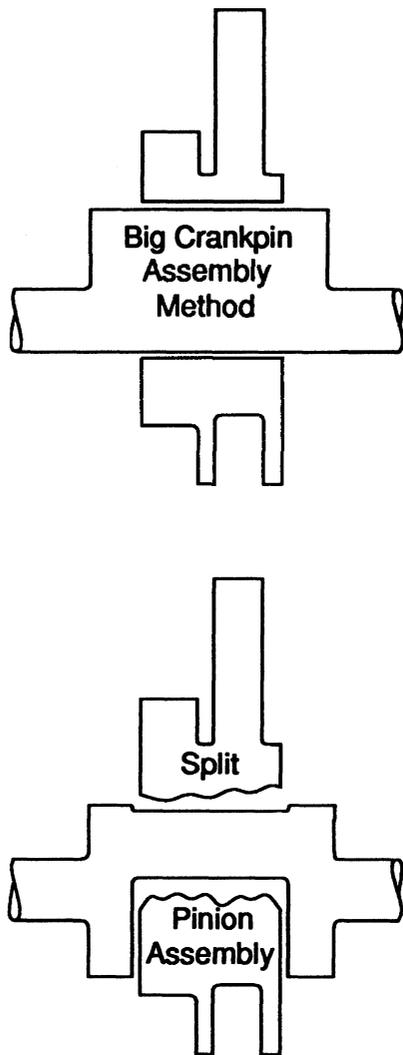


Figure 13 - Alternative Assembly Methods

An alternative to the split pinion assembly that was considered can be termed the "big crankpin" design, and is found in the patent by Huf (20). A representation of this design is found in Figure 13. By increasing the crankpin diameter as shown in the figure, the pinion assembly can be slid on and off without separation. The primary disadvantage to this design alternative is the resulting large size of engine components for a set stroke, or the necessity of reducing engine stroke to maintain compact component size. Additionally, the cylindrical crankweb/bearings discussed in the previous section cannot be used with the "big crankpin" design without resulting in very large components.

Elimination of the alternative design concepts for pinion assembly installation led to the decision to split the crankshaft in some fashion. Slider-crank engines sometimes use two-piece crankshafts in which the crankpin in

one half is press fit into the web in the other half, as shown in Figure 14. This is especially common for motorcycle engines, and was used for the Honda SL90. To perform maintenance on the pinion assembly, the crankshaft press fit would need to be periodically separated and then re-pressed. It was unclear how a traditional press fit would fare under this repeated stressing, especially with the frequent assembly and disassembly expected with prototype work. As a result, alternatives to this assembly method were considered.

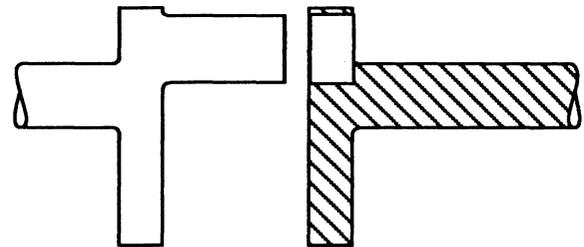


Figure 14 - Conventional Press Fit Crankshaft Assembly

The design alternative developed for crankshaft assembly and disassembly is seen in Figure 15.

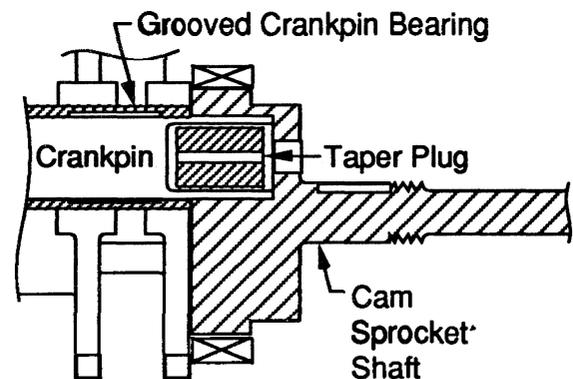


Figure 15 - Modified Hypocycloid Engine Crankshaft Assembly

The crankpin is ground on its outer diameter to provide approximately zero clearance with the accompanying hole in the cam sprocket shaft. A standard taper reamer is used to ream a tapered hole in the output shaft. Prior to assembly, a crankpin taper plug (constructed from a standard steel taper pin) is installed finger-tight in the tapered hole. The cam sprocket shaft is then slid onto the crankpin using an alignment jig for correct axial spacing and angular positioning. The crankpin taper plug is forced into the

tapered hole using a steel rod inserted through an access hole in the cam sprocket shaft.

To remove the plug, first grease is used to fill the clearance space between the end of the plug and the bottom of the taper hole in the crankpin. A pin acting as a ram is then inserted into the hole through the center of the taper plug and struck sharply, driving out the plug with high pressure grease. Alternatively, the hole through the center of the taper plug could be threaded and a jackscrew used to drive the taper pin out of the hole.

There are several advantages to this assembly method. The entire length of the cam sprocket shaft hole acts as guide surface for the crankpin, ensuring that the crankpin is perpendicular to the cam sprocket shaft face. Additionally, little relative motion is required between any of the components. By pushing the crankpin in as little as 2 mm or less, a tight interference fit and therefore a large torque capacity is developed. Reduced relative motion between the components results in less wear.

**GROOVED CRANKPIN BEARING** - One further feature designed to reduce crankshaft bending moments involves the crankpin bearing, which supports the pinion assembly. As seen in Figure 15, the one-piece journal bearing is grooved. This groove distributes loads from the piston-rod to the ends of the crankpin and closer to the main bearings, reducing crankpin bending loads.

**PLANET GEAR BACKLASH ADJUSTMENT** - A variety of potential advantages provided by hypocycloid engines and the modified hypocycloid engine were previously described. Many of these advantages depend heavily on straight-line motion of the piston-rod, parallel to and coincident with the cylinder centerline. Figure 9 depicts straight-line motion for the modified hypocycloid engine. Point  $D_1$ , center of the piston-rod big end, dictates the motion of the piston-rod.  $D_1$  will remain on the vertical cylinder centerline provided that the lengths of links  $L_1$  and  $L_2$  remain the same and that angles  $\theta_1$  and  $\theta_2$  remain equal. However, if all backlash is not taken up in the gear meshes while  $D_1$  is on the vertical centerline, the pinion and link  $L_2$  will rotate with respect to link  $L_1$ , the crankshaft. Point  $D_1$  will still describe a straight-line, but inclined to the vertical cylinder centerline. The resulting angular oscillation for a one-piece piston-rod may cause jamming of the piston in the cylinder, resulting in high friction forces and

piston-rod bending. Even if the gear set is adjusted so that straight-line vertical motion occurs for the stroke in one direction (eg., from top dead center to bottom dead center), when the stroke changes to the opposite direction, the backlash in the gearing will cause the piston motion to deviate from the vertical direction. This is because of a torque reversal that can be explained with the assistance of Figure 10. As the pinion gear passes through either of the dead center positions, the torque on the pinion due to the gas load  $F_g$  changes direction. Assuming a constant pinion rotational speed  $2\omega_c$  about the crankpin, the direction of the pinion gear tangential tooth load,  $W_t'$ , must also change. Similarly, tooth loads on the ring, planet and sun gears must change assuming constant rotational speeds for these gears. Gear backlash takeup would depend on the dynamics of the particular hypocycloid train under consideration, but the potential for deviations from vertical straight-line motion and subsequent piston jamming are evident. The potential piston motion is further illustrated by Figure 16.

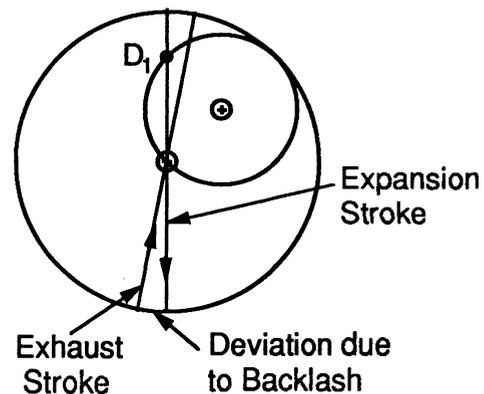


Figure 16 - Gear Backlash Effects in the Hypocycloid Engine

In order to exercise some control over the straight-line motion in the modified hypocycloid engine, an adjustment feature was introduced for the prototype design. Referring to Figure 11, the shafts for the planet gears are press fit into the ring gear support {53}. The ring gear support is mounted with a slip-type fit into a cylindrical bore in the crankcase {24}. By rotating the ring gear support with respect to the crankcase, backlash in the entire gear train can be taken up as desired. This allows some control over the motion of point  $D_1$  in Figure 9. For example, the ring gear support might be adjusted so that point

$D_1$  would follow the vertical centerline on the expansion stroke of the engine. Alternatively, the ring gear support might be adjusted so that point  $D_1$  follows a slightly inclined line on both the expansion and compression strokes. The best adjustment would be determined by work with the prototype engine.

### CONSTRUCTION OF THE PROTOTYPE MODIFIED HYPOCYCLOID ENGINE

The benefits provided by the basic hypocycloid engine and additional advantages offered by the modified hypocycloid engine are only valuable if they can be realized in an actual engine. Currently, a prototype of the modified hypocycloid engine is being constructed. This engine will be used in a test program designed to verify these benefits and to evaluate the overall design.

A variety of considerations are involved in the translation of the design from paper to working engine. One of these is manufacturing and assembly tolerances and how these tolerances will affect straight-line piston motion and engine performance. Some work has been done which addresses performance in hypocycloid engines as affected by manufacturing tolerances. Ishida has performed an analysis showing that residual unbalance due to manufacturing errors for a hypocycloid engine are proportional to the angular oscillation of the connecting rod, which is ideally non-existent. Ishida provides experimental data from a hypocycloid engine showing maximum angular oscillations of only 0.17 degrees, and states that this oscillation did not act as a significant vibrating force in the engine (23). These results suggest that it is reasonable to expect significant improvements in vibration for the modified hypocycloid engine. Additionally, small angular oscillations of the piston-rod should not prevent effective sealing of the cylinder block. Ishida did not use a one-piece piston-rod in his work, however, as incorporated in our modified hypocycloid engine design. As discussed in the previous section, this one-piece design might lead to piston jamming, high piston friction forces, and piston-rod bending, even with small piston-rod angular oscillation. These concerns will be investigated in the prototype engine.

It is felt that several other concerns are best addressed through construction of a prototype

engine. These include manufacturability and cost of the engine, which will be considered during engine construction. The completed engine can also serve as a test vehicle for investigating piston slap and emissions in a hypocycloid engine.

### SUMMARY AND CONCLUSIONS

The hypocycloid engine has been presented as a concept which provides many advantages over the slider-crank engine. These include perfect balance, the reduction of piston assembly friction and piston slap, and the potential for application in adiabatic engines. The hypocycloid engine also has been shown to eliminate linear bearing friction and bending loads on the piston-rod which occur in other sinusoidal engines. The modified hypocycloid engine incorporates the advantages of the basic hypocycloid engine while providing reduced gear and crankshaft gas loads, making it a particularly attractive candidate for supercharged and other high gas pressure applications.

The design of a prototype modified hypocycloid engine has been described. Details of the engine sub-assemblies and patented design characteristics of the engine have been included. These design characteristics reduce crankshaft loading, facilitate crankshaft assembly and disassembly, and provide for backlash adjustment in the modified hypocycloid gear train. Many of these characteristics are equally applicable to basic hypocycloid engine design.

A prototype modified hypocycloid engine is currently under construction. This engine will be used in a test program to verify the general engine concept and to evaluate the proposed advantages offered by the design.

### NOMENCLATURE

- C - location of crankpin axis
- $D_1$  - point on hypocycloid engine undergoing straight-line vertical motion, to which piston-rod is connected
- $D_2$  - point on hypocycloid engine undergoing straight-line horizontal motion perpendicular to  $D_1$
- $D_p$  - pinion pitch diameter, modified hypocycloid engine

E	- location of crankshaft counter-weights
$F_g$	- gas force on the piston
$F_t$	- tangential load from the crankpin on the basic hypocycloid engine pinion
$F_t'$	- tangential load from the crankpin on the modified hypocycloid engine pinion
L	- engine stroke
$L_1, L_2$	- hypocycloid mechanism links equal to 1/4 the engine stroke
O	- location of crankshaft axis
$P_i$	- ideal power transmitted through hypocycloid gearing
$(P_{eq})$	- equivalent power transmitted by an individual gear mesh
T	- torque on the basic hypocycloid engine crankshaft
$T'$	- torque on the modified hypocycloid engine crankshaft
$T'_{crankpin}$	- torque on the modified hypocycloid crankshaft through the crankpin
$T'_{sun}$	- torque on the modified hypocycloid crankshaft through the sun
V	- pitch-line velocity of tooth engagement at a gear mesh.
$W_t$	- tangential gear tooth load on basic hypocycloid engine pinion
$W_t'$	- tangential gear tooth load on modified hypocycloid engine pinion
$\phi$	- % power loss of an individual gear mesh
$\eta$	- basic hypocycloid engine gearing efficiency
$\eta'$	- modified hypocycloid engine gearing efficiency
$\theta_c$	- engine crankangle
$\omega_c$	- crankshaft angular velocity

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